BABA BANDA SINGH BAHADUR ENGINEERING COLLEGE

Department of Applied Sciences

Question Bank

Semester: First **Subject:** Mathematics Paper-I **Code:** BTAM104-18

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1. State and prove Rolle's theorem.
2. State Lagrange's Mean Value theorem.
3. State and prove Cauchy's Mean Value theorem.
4. Verify Rolle's theorem for the following functions:

   (i) $f(x)=x(x-2)e^{\frac{3x}{4}}$ in $(0,2)$

   (ii) $f(x)=x^{2m-1}(a-x)^{2n}$ in $(0,a)$

   (iii) $f(x)=\frac{\sin x}{e^x}$ in $[0,\pi]$

   (iv) $f(x)=2+(x-1)^{\frac{2}{3}}$ in $[0,2]$

   [Hint: Not differentiable at x = 1]
5. Find a root (solution) of the equation $x\ln x-2+x=0$ lying in (1, 2).
6. Show that the polynomial equation $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+.........+a_n=0$ has at least one real root in (0, 1) if $\frac{a_0}{n+1}+\frac{a_1}{n}+..........+\frac{a_{n-2}}{2}+a_n=0$ and $a_0,a_1,.......a_n$ are real numbers.

   [Hint: Take $f(x)=\frac{a_0}{n+1}x^{n+1}+\frac{a_1}{n}x^n+..........+\frac{a_{n-2}}{2}x^2+a_nx$ in [0, 1]. Apply Rolle's theorem]
7. Deduce Lagrange's Mean Value theorem form Rolle's theorem.

   [Hint: Choose $g(x)=f(x)-f(a)-A(x-a)$, $g(a)=0$, determine A such that $g(b)=0$, $g(x)$ satisfies all the conditions of Rolle's theorem]

8. Show that $\frac{h}{\tan^{-1} h} < \tan^{-1} h < h$ when $h \neq 0$ and $h > 0$.

   [Hint: Take $f(x) = \tan^{-1} x$ in $0 \leq x \leq h$]

9. Calculate approximately $\sqrt[5]{245}$ by using LMV theorem.

   [**Another form of LMV theorem:**

   We know that $\frac{f(b) - f(a)}{b-a} = f'(a)$, Take $b = a + h$, then we get

   $\frac{f(a+h) - f(a)}{a+h-a} = f'(a+\theta h)$ where $c = a + \theta h$ lies between a and $b = a + h$ when $0 < \theta < 1$. Thus, $f(a+h) = f(a) + hf'(a+\theta h);\ 0 < \theta < 1$

   Hint: Use another form of LMV theorem *i.e.* $f(a+h) = f(a) + hf'(c)$, here choose $f(x) = x^{\frac{1}{5}}, a = 243, b = 245$ and $c = 243$ approximately]

10. Calculate approximately $\sqrt[6]{65}$ by using LMV theorem.

11. Let $f(x)$ be continuous on $[a-1, a+1]$ and differentiable in $(a-1, a+1)$. Show that there exists a $\theta, 0 < \theta < 1$ such that

$$f(a-1) - 2f(a) + f(a+1) = f'(a+\theta) - f'(a-\theta).$$

   [Hint: Define $\phi(t) = f(a+t) + f(a-t)$, Apply LMV theorem on $[0,1]$]

12. Use LMV theorem to prove that if $0 < u < v$, $\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v+u}{1+v^2}$. Also, deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$. [Take $f(x) = \tan^{-1} x,\ u < x < v$]

13. Verify Cauchy's Mean Value theorem for the functions:

   (i) $f(x) = x^4, g(x) = x^2$ in the interval [a, b]

   (ii) $f(x) = \ln x, g(x) = \frac{1}{x}$ in the interval [1, e]

   (iii) $f(x) = e^x, g(x) = e^{-x}$ in the interval (a, b)

   (iv) $f(x) = \sqrt{x}, g(x) = \frac{1}{\sqrt{x}}$ in the interval (a, b)

   (v) $f(x) = \frac{1}{x^2}, g(x) = \frac{1}{x}$ in the interval (a, b)

   (vi) $f(x) = x^3 - 3x^2 + 2x, g(x) = x^3 - 5x^2 + 6x$ in the interval (*0, 0.5*)

14. If $0 \le a < b < \frac{\pi}{2}$, Show that $0 < \cos a - \cos b < (b-a)$.

15. Show that there exists a number $c \in (a,b)$ such that $2c\left[f(a)-f(b)\right] = f'(c)\left[a^2-b^2\right]$ when f is continuous in [*a*, *b*] and derivable in (*a*, *b*).

    [Hint: Apply CMV theorem to the two function *f(x)* and *g(x)* = x^2]

16. Show that $\sin b - \sin a < b - a$ if $0 < a < b < \frac{\pi}{2}$.

    [Hint: Take $f(x) = \sin x,\ g(x) = x$ in CMV theorem]

17. Define improper integral of first kind.
18. Discuss the convergence of beta function.
19. Discuss the convergence of the integral $\int_1^\infty \frac{1}{x} dx$.
20. Discuss the convergence of the integral $\int_1^\infty \frac{1}{(1+x)\sqrt{x}}\, dx$.
21. Discuss the convergence of the integral $\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$.
22. Discuss the convergence of the integral $\int_{-\infty}^\infty \frac{1}{x^2+2x+2}\, dx$.
23. Discuss the convergence of the integral $\int_0^4 \frac{1}{x(4-x)}\, dx$.
24. Discuss the convergence of the integral $\int_0^2 \frac{1}{2x-x^2}\, dx$.
25. Discuss the convergence of the integral $\int_0^\infty e^{2x}\, dx$.
26. Discuss the convergence of the integral $\int_0^\pi \frac{1}{1+\cos x}\, dx$.
27. Prove that $\Gamma(1) = 1$.
28. Prove that $\Gamma(n) = (n-1)\Gamma(n-1)$.
29. Prove that $\Gamma(n) = (n-1)!$, where *n* is a positive integer.

30. Show that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

31. Evaluate the following:

(i) $\int_0^\infty \sqrt[4]{x}.e^{-\sqrt{x}}dx$

(ii) $\int_0^\infty \sqrt{x}.e^{-\sqrt[3]{x}}dx$

(iii) $\int_0^\infty x^{n-1}.e^{-h^2x^2}dx$

(iv) $\int_0^\infty \frac{x^a}{a^x}.dx$

(v) $\int_0^1 x^{n-1}\left[\log_e\left(\frac{1}{x}\right)\right]^{m-1}.dx$

(vi) $\int_0^\infty x^4.e^{-x^2}dx$

(vii) $\int_0^1 (x\log x)^3.dx$

(viii) $\int_0^1 \frac{1}{\sqrt{x\log\frac{1}{x}}}.dx$

(ix) $\int_0^1 x^4\left(1-\sqrt{x}\right)^5.dx$

(x) $\int_0^\infty \frac{x^4\left(1-x^5\right)}{(1+x)^{15}}.dx$

32. Prove that $\int_0^1 x^m(\log x)^n.dx=\frac{(-1)^n}{(m+1)^{n+1}}\Gamma(n+1)$

33. Prove that $\int_0^\infty e^{-ky}y^{n-1}.dy=\frac{1}{k^n}\Gamma(n)$

34. Prove that $\int_0^1\left(\log\frac{1}{y}\right)^{n-1}.dy=\Gamma(n)$

35. Prove that $\frac{1}{n}\int_0^\infty e^{-x^{\frac{1}{n}}}dx=\Gamma(n)$

36. Prove that $1.3.5........(2n-1)=\frac{2^n}{\sqrt{\pi}}\Gamma\left(n+\frac{1}{2}\right).$

37. Show that $\int_0^{\frac{\pi}{2}}\sin^p\theta\cos^q\theta.d\theta=\frac{1}{2}\frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{\Gamma\left(\frac{p+q+2}{2}\right)}$.

38. Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.

39. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}.d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$.

40. Using Beta and Gamma functions, evaluate $\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{\frac{1}{2}}.dx$

41. Using Beta and Gamma functions, evaluate $\int_0^\infty \frac{dx}{1+x^4}$.

42. Assuming $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}, 0 < n < 1$, show that

$$\int_0^\infty \frac{x^{p-1}}{1+x}\,dx = \frac{\pi}{\sin p\pi}; \qquad 0 < p < 1$$

43. Prove that $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \pi\sqrt{2}$.

44. Prove that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

[This result is known as **Duplication formula**]

45. Show that $\int_0^\infty x^2 e^{-x^4}.dx \;\; \text{x} \;\; \int_0^\infty \frac{e^{-x^4}}{\sqrt{x}}.dx = \frac{\pi}{4\sqrt{2}}$.

46. Show that:

(vii) $\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^4\theta.d\theta = \frac{\pi}{32}$

(viii) $\int_0^{\frac{\pi}{2}} \sin^3 x\cos^{\frac{5}{2}} x\,dx = \frac{8}{77}$

47. Prove that $\int_3^7 \sqrt[4]{(x-3)(7-x)}dx = \frac{2\left(\Gamma\left(\frac{1}{4}\right)\right)^2}{3\sqrt{\pi}}$.

[Hint: Put $x = 4t+3$]

48. Prove that $\int_0^b (x-a)^m (b-x)^n\,dx = (b-a)^{m+n-1}\,\beta(m+1,n+1)$

49. Find the volume of the sphere of radius *a*.

50. Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the x-axis.

51. Find the surface of the solid generated by the revolution of the ellipse $x^2+4y^2=16$ about its major axis.

52. Find the surface of the solid generated by the revolution of the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ about the x-axis.

53. The curve $r=a(1+\cos\theta)$ revolves about the initial line. Find the surface of the figure so obtained.

54. If $A=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$, then for what value of θ is A an identity matrix?

55. Construct a mxn matrix $A=\left[a_{ij}\right]$, where $a_{ij}=\frac{|2i-3j|}{2}$; $m=2, n=2$.

56. If $\begin{pmatrix}2x+1 & 2y\\ 0 & y^2+1\end{pmatrix}=\begin{pmatrix}x+3 & 10\\ 0 & 26\end{pmatrix}$, write the value of $y+x$.

57. Find non-zero values of x such that: $x\begin{pmatrix}2x & 2\\ 3 & x\end{pmatrix}+2\begin{pmatrix}8 & 5x\\ 4 & 4x\end{pmatrix}=2\begin{pmatrix}x^2+8 & 24\\ 10 & 6x\end{pmatrix}$.

58. Assume that *Y*, *W* and *P* are matrices of order 3 x *k*, *n* x 3 and *p* x *k* respectively. Find the restrictions on *n*, *p*, *k* so that *PY* + *WY* is defined.

59. If $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}3 & 1\\ 2 & 5\end{pmatrix}=\begin{pmatrix}7 & 11\\ k & 23\end{pmatrix}$, then write the value of *k*.

60. A matrix *A* has *a* + *b* rows and a + 2 columns while the matrix *B* has *b* + 1 rows and *a* + 3 columns. Both matrices *AB* and *BA* exist. Find *a* and *b*. Can you say *AB* and *BA* are of same type? Are they equal?

61. If *A* and *B* are square matrices of same order and *k* is any scalar, prove that *A*-*k I* and *B*-*k I* commute if and only if *A* and *B* commute.

62. Give an example of two matrices A and B such that $AB=O$, where neither $A=O$ nor $B=O$.

63. If *A* is a matrix of order 2 x 3 and *B* is a matrix of order 3 x 5, what is the order of the matrix $(AB)^T$.

64. For what value of k, the matrix $\begin{pmatrix}2-k & 3\\ -5 & 1\end{pmatrix}$ is not invertible?

65. Solve the following system of linear equations by matrix inversion method:

(i) $x+y+z=8, \quad x-y+2z=6, \quad 3x+5y-7z=14$

(ii) $x+2y+3z=1, \quad 2x+3y+2z=2, \quad 3x+3y+4z=1$

66. Solve the following system of linear equations by Cramer's rule:

(i) $2x+3y=5, \quad 11x-5y=6$

(ii) $x+y+z=6, \quad x-y+2z=5, \quad 3x+y+z=8$

(iii) $x+2y+3z=1, \quad 2x+3y+2z=2, \quad 3x+3y+4z=1$

(iv) $2x-y+3z=9, \quad y-z=-1, \quad x+y-z=0$

67. Explain elementary transformations on a matrix.

68. If $A=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ then show that $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$. Hence find A^{50}.

69. Compute the inverse of the following matrices by Gauss Jordan method (Elementary Row transformations):

(i) $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ *(iv)* $\begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$

70. Define rank of a matrix and give one example. What is the rank of a

(a) Singular matrix of order n?

(b) Non-singular matrix of order n?

71. Find the rank of the following matrices:

(i) $\begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & 3 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$

72. Reduce the following matrices to normal form and hence find rank:

(i) $\begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$ *(iii)* $\begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{pmatrix}$

(iv) $\begin{pmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{pmatrix}$ *(v)* $\begin{pmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 3 & 4 & 3 \end{pmatrix}$

73. If $A=\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{pmatrix}$; find two non-singular matrices P and Q such that PAQ is in the normal form. Also find A^{-1} (if it exists).

74. Solve the following system of linear equations by Gauss Elimination method:

(i) $x+y+z=6,\quad x-y+2z=5,\quad 3x+y+z=8$

(ii) $x+2y+3z=1,\quad 2x+3y+2z=2,\quad 3x+3y+4z=1$

75. Solve the following system of linear equations by Gauss Jordan method:

(i) $x+y+z=8,\quad x-y+2z=6,\quad 3x+5y-7z=14$

(ii) $x+2y+3z=1,\quad 2x+3y+2z=2,\quad 3x+3y+4z=1$

76. State the conditions in terms of rank of the coefficient matrix and rank of the augmented matrix for *(a)* Unique solution *(b)* No solution *(c)* Infinite many solution.

77. Investigate for consistency of the following equations and if possible find the solutions:

$$4x-2y+6z=8,\quad x+y-3z=-1,\quad 15x-3y+9z=21$$

78. Show that the equations $2x+6y+11=0,\quad 6x+20y-6z+3=0,\quad 6y-18z+1=0$ are not consistent.

79. For what values of λ and μ do the system of equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have *(i)* No solution *(ii)* A unique solution *(iii)* An infinite number of solutions.

80. Find the real value of p for which the system of equations $x+2y+3z=px,\quad 3x+y+2z=py,\quad 2x+3y+z=pz$ have non-trivial solution.

81. For what value (s) of k, the equations $x+y+z=1, 2x+y+4z=k, 4x+y+10z=k^2$ have a solution? Solve them completely in each case.

82. Investigate the value of λ and μ so that the equations $2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$ have *(i)* No solution *(ii)* A unique solution *(iii)* An infinite number of solutions.
83. Test the following system of equations for consistency and solve
    $x+2y+z=3,\ \ 2x+3y+2z=5,\ \ 3x-5y+5z=2,\ \ 3x+9y-z=4$
84. Show that the equations $3x+4y+5z=a,\ \ 4x+5y+6z=b,\ \ 5x+6y+7z=c$ do not have a solution unless $a+c=2b$.
85. For what value of k the system of equations $x+y+z=2,\ \ x+2y+z=-2,$ $x+y+(k-5)z=k$ has no solution?
86. For what value (s) of k, do the vectors $(k,1,1),(0,1,1),(k,0,k)$ are linearly independent.
87. Test whether the subset S of R^3 is L.I. or L.D. , given S = {(1,0,1), (1,1,0), (-1,0,-1)}
88. Define linear dependence of vectors and determine whether the vectors $(3,2,4),(1,0,2),(1,-1,-1)$ are linearly dependent or not?

Discuss whether V defined in problems 89 to 94 is a vector space or not. If V is not a vector space, state which of the properties are not satisfied.

89. Let V be the set of al real polynomials of degree 4 or 6 with usual addition and scalar multiplication.
90. Let V be the set of all rational numbers with the usual addition and scalar multiplication.
91. Let V be the set of all positive real numbers with addition defined as $x+y=xy$ and usual scalar multiplication.
92. Let V be the set of all ordered (x,y) pairs in $\mathbb{R}^2$ with vector addition defined as $(x,y)+(u,v)=(x+u,y+v)$ and scalar multiplication defined as $\alpha(x,y)=(3\alpha x,y)$.
93. Let V be a set of all real valued continuous functions *f* on [*a*, *b*] such that $\int_a^b f(x)\,dx=0$ with usual addition and scalar multiplication.
94. Let V be a set of all real valued continuous functions *f* on [*a*, *b*] such that $\int_a^b f(x)\,dx=2$ with usual addition and scalar multiplication.

Is W a vector subspace of V in problems 95- 97? If not, state why?

95. Let V be the set of all 3 x 1 real matrices with usual addition and scalar multiplication and W consisting of all 3 x 1 real matrices of the form $\begin{bmatrix} a \\ b \\ a+b \end{bmatrix}$.

96. Let V be the set of all 3 x 3 real matrices with usual matrix addition and scalar multiplication and W consisting of all 3 x 3 real matrices A which
    (i) have positive elements
    (ii) are symmetric

97. Let V be thee vector space of all the triplets of the form (x, y, z) in $\mathbb{R}^3$ with usual addition and scalar multiplication and W be the set of all triplets (x, y, z) of the form such that
    (i) $x = 2y = 3z$
    (ii) $x \geq 0,\ \ y, z$ arbitrary.
    (iii) z is an integer.

98. Check whether the following subsets of R^3 are subspaces or not
    (i) $\{(a,b,c) : b = c = 0\}$
    (ii) $\{(a,b,c) : c = 3\}$
    (iii) $\{(x, y, z) : x + y + z = 1\}$
    (iv) $\{(a,b,c) : b = 3a + 2\}$

99. Show that for the set $S \in R_{2X2}$, where $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + b = 0 \right\}$, S is a subspace of R_{2X2}.

100. Prove that intersection of two subspaces of a vector space is a subspace.

101. Give an example to show that union of two subspaces of a vector space need not to be a subspace.

102. Express (5, 9) as a linear combination of the vectors (1, 2) and (2, 3).

103. Show that the vector (0, 2, 1) cannot be expressed as a linear combination of the vectors (1, 1, 2) and (2, 2, 4).

104. Show that the set of matrices $\left\{ \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix} \right\}$ in $M_2(R)$ is linearly independent.

105. Determine whether the vector (3, 2, 5) is a linear combination of the vectors $(1,2,-1),(2,3,4)$ and $(1,5,-3)$.

106. Let $u=(1,-2,1,3), v=(1,2,-1,1)$ and $(2,3,1,-1)$. Determine whether or not x is a linear combination of u, v w , where x is given by

(i) $(2,-7,1,11)$

(ii) $(4,3,0,3)$

107. Prove that the vectors $x=(1+i,2i)$, $y=(1,1+i)$ in $V_2(C)$ are L.D. but L.I. in $V_2(R)$.

108. Let V(F) be vector space. Show that the vectors $\alpha u,\ \beta v,\ \alpha u+\beta v$ are L.D.

109. Show that the set $S=\left\{\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\-1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$ spans V, where V is the set of all 3 x 1 real matrices.

110. Let V be the set of all 2 x 2 real matrices. Show that the set $S=\left\{\begin{pmatrix}2&1\\1&-2\end{pmatrix},\begin{pmatrix}1&1\\1&0\end{pmatrix},\begin{pmatrix}0&0\\1&1\end{pmatrix},\begin{pmatrix}0&2\\0&-1\end{pmatrix}\right\}$ spans V.

111. Let $P(y)=y^2-4y-6, Q(y)=2y^2-7y-8, R(y)=2y-3$.Write $S(y)=2y^2-3y-25$ as a linear combination of $P(y),Q(y),R(y)$.

112. If x, y, z are linearly independent vectors in $\mathbb{R}^3$, then show that

(i) $x + y , y + z, z + x$

(ii) $x , x + y, x + y + z$

are also linearly independent in $\mathbb{R}^3$.

113. Let V be a vector space. Then show that

(i) Set $\{v\}$ is L.D. iff $v = 0$

(ii) Set $\{v_1,v_2\}$ is L.D. iff v_1 and v_2 are collinear *i.e.* one of them is a scalar multiple of other.

114. In a vector space V(F), show that

(i) Any subset of L.I. set is L.I.

(ii) Any superset of L.D. set is L.D.

115. Show that the set of vectors is linearly dependent if and only if at least one element of the set is a linear combination of the remaining elements.
116. Define Symmetric matrix. Also give an example.
117. Define Skew-symmetric matrix. Also give an example.
118. For what value of k, the matrix $\begin{pmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{pmatrix}$ is skew- symmetric ?
119. Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.
120. Show that the elements on the main diagonal of a skew symmetric matrix are all zero.
121. If a matrix A is symmetric as well as skew symmetric, then show that $A = O$.
122. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute.
123. Express the matrix $\begin{pmatrix} 14 & 17 & 18 \\ 19 & 6 & -7 \\ 1 & 2 & 5 \end{pmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.
124. Define orthogonal matrix. Also give an example.
125. Show that the product of two orthogonal matrices of same order is also an orthogonal matrix.
126. Show that transpose of an orthogonal matrix is also orthogonal.
127. If A be an orthogonal matrix, show that $|A| = \pm 1$.
128. Verify that the matrix $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$ is orthogonal.
129. If $\langle l_i, m_i, n_i \rangle$, $i = 1,2,3$ are the direction cosines of three mutually perpendicular lines referred to an orthogonal Cartesian coordinate system, prove that the matrix $\begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix}$ is orthogonal.

130. If A is symmetric and P is orthogonal, show that $P^{-1}AP$ is symmetric.

131. Show that at least one latent root of every singular matrix is zero.

132. Show that, if zero is an eigen value of a matrix then it is singular.

133. Show that a square matrix and its transpose have the same set of eigen values.

134. If λ is an eigen value of square matrix A, then show that λ^m is an eigen value of $A^m \;\forall\; m \in N$.

135. If A is a non-singular matrix, prove that the eigen value of A^{-1} are the reciprocal of the eigen values of A.

136. Define similar matrices and prove that similar matrices have same characteristic roots.

137. If $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$, find the eigen values of A^4.

138. The characteristic roots of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & k & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 0, 3, 15. Find the value of k.

139. Determine the eigen values and corresponding eigen values of the following matrices:

(i) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ *(iii)* $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

(iv) $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ *(v)* $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ *(vi)* $\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -7 & 5 & 1 \end{pmatrix}$

140. Diagonalize the matrix $\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$.

141. Diagonalize, if possible, the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & -1 & 4 \end{pmatrix}$.

142. Diagonalize the following matrices:

(i) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$

143. Diagonalize $A=\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and hence find A^8.

144. If $A=\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$, find the eigen values of A^{-1}.

145. Show that inverse of an orthogonal matrix is also orthogonal.