

# Heat Transfer

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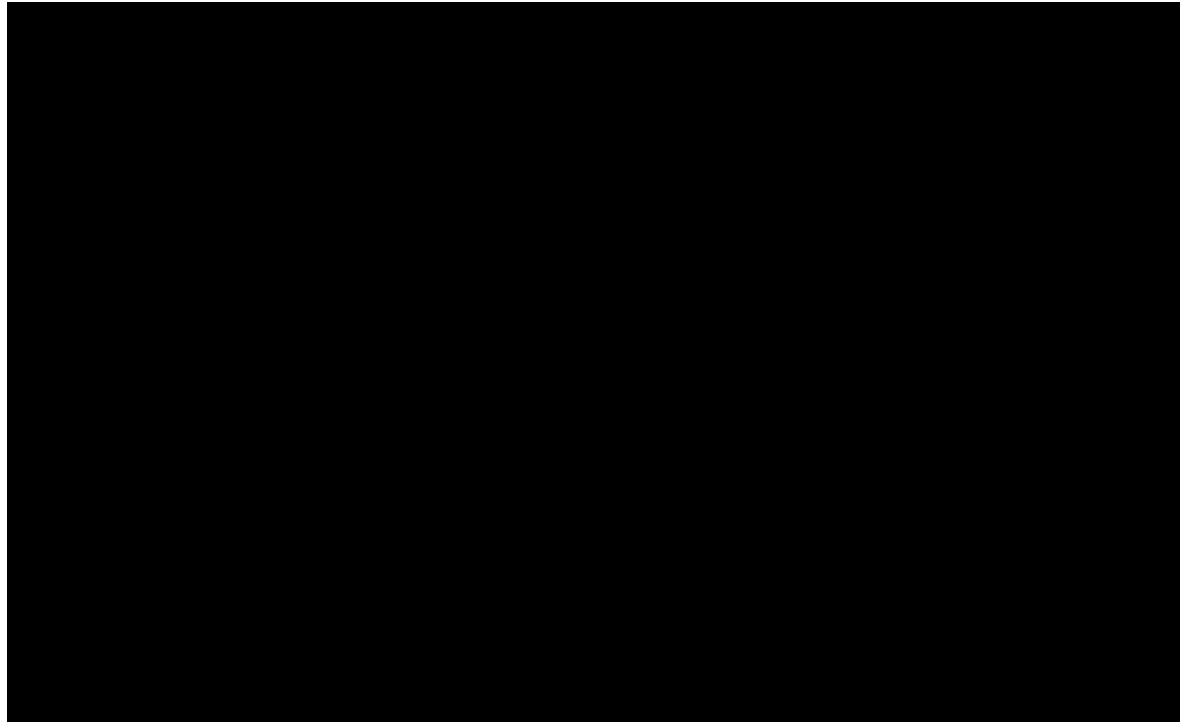
- Heat always moves from a warmer place to a cooler place.
- Hot objects in a cooler room will heat up to room temperature.
- Cold objects in a warmer room will cool to room temperature.

# Modes of Heat Transfer

- Heat transfers in three ways:
  - Conduction
  - Convection
  - Radiation

# Conduction

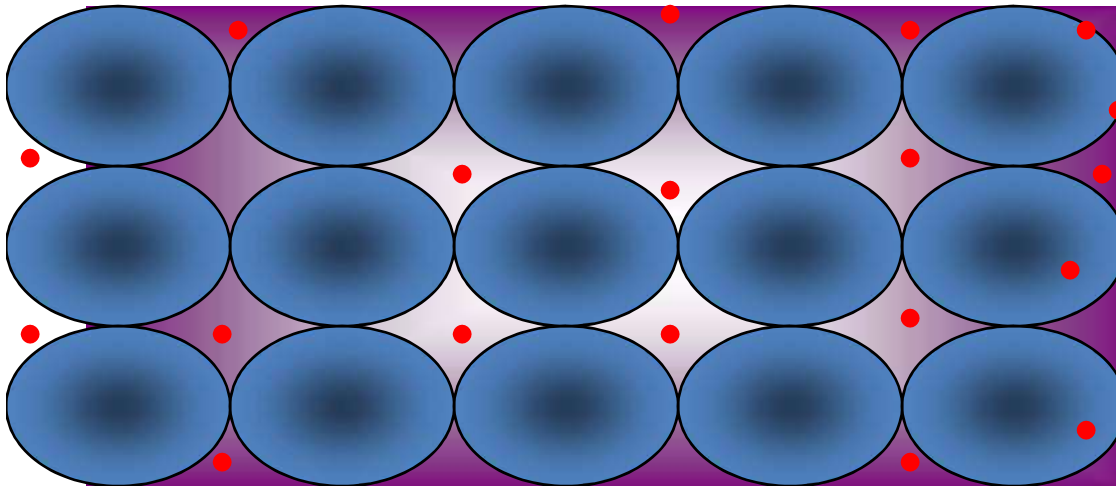
Conduction is a mode of heat transfer which mainly take place in **solids**. When we heat a metal strip at one end, the heat travels to the other end.



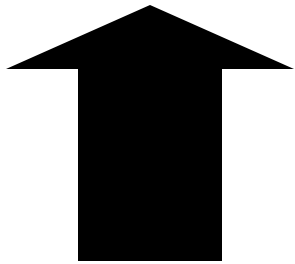
As we heat the metal, the particles vibrate, these vibrations make the adjacent particles vibrate, and so on and so on, the vibrations are passed along the metal and so is the heat. We call this conduction.

# Conduction in Metals are different

The outer electrons of metal atoms are free to move.



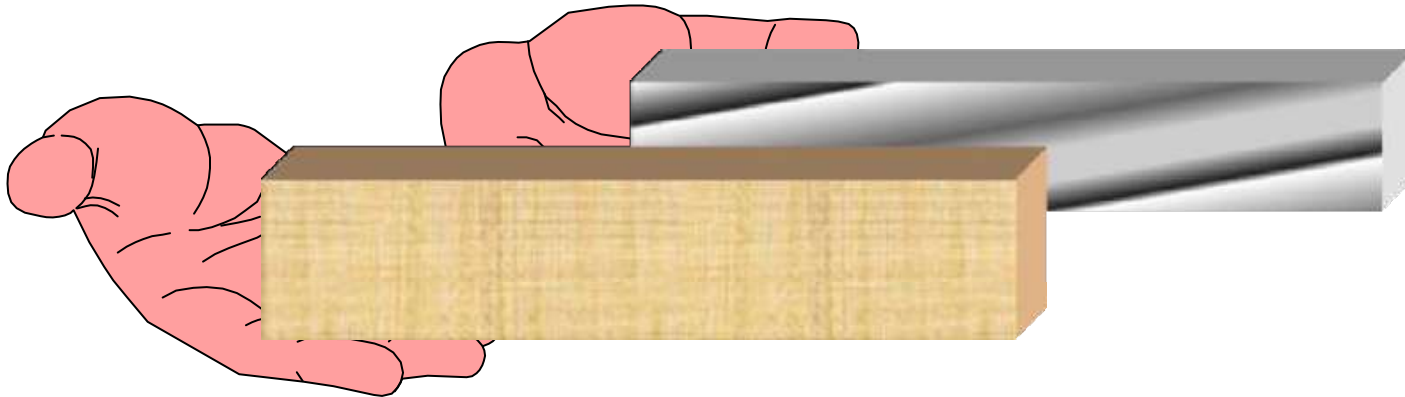
When the metal is heated, this 'sea of electrons' gain kinetic energy and transfer it throughout the metal.



Insulators, such as **wood** and **plastic** do not have this 'sea of electrons' which is why they do not conduct heat as well as in metals.

# Why does metal feel colder than wood, if they are both at the same temperature?

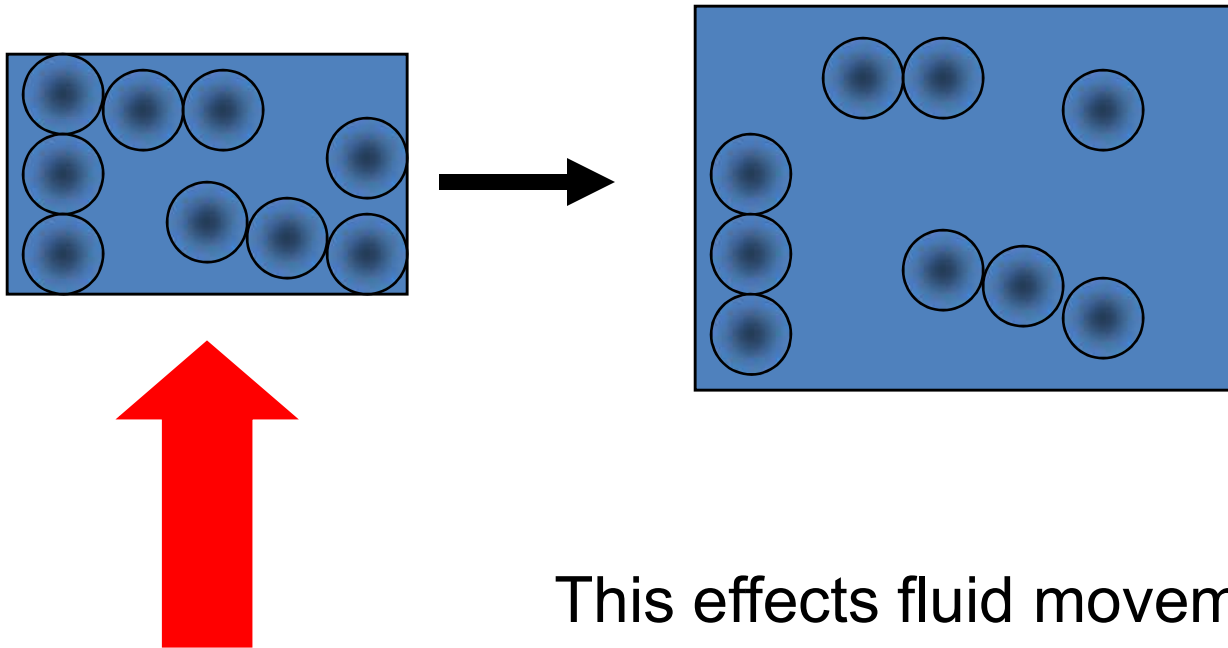
Metal is a conductor, wood is an insulator. Metal conducts the heat away from our hands. Wood does not conduct the heat away from our hands as well as the metal, so the wood feels warmer than the metal.



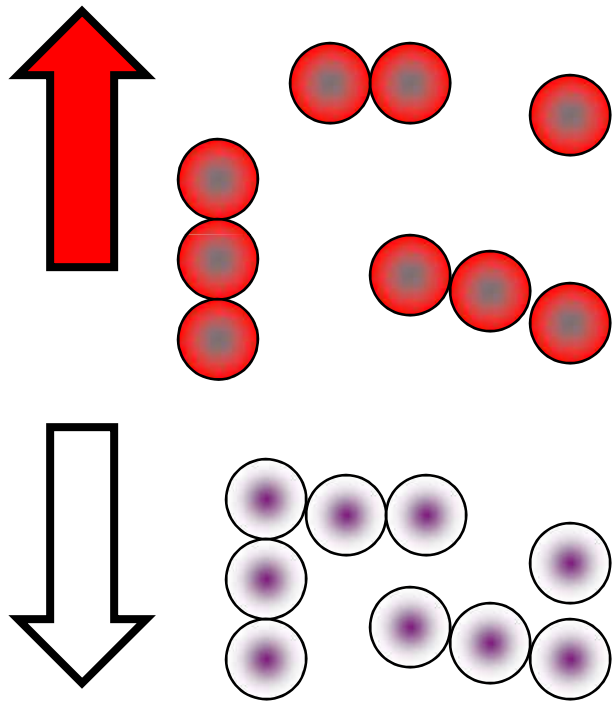
# Convection

This is the mode of heat transfer which take place in **fluids**.  
What happens to the particles in a liquid or a gas when we heat them?

The particles spread out and become less dense.



# Fluid movement



Cooler(more dense) fluids sink through warmer(less dense) fluids.

In other words, warmer fluid particles rise up and cooler particles sink.

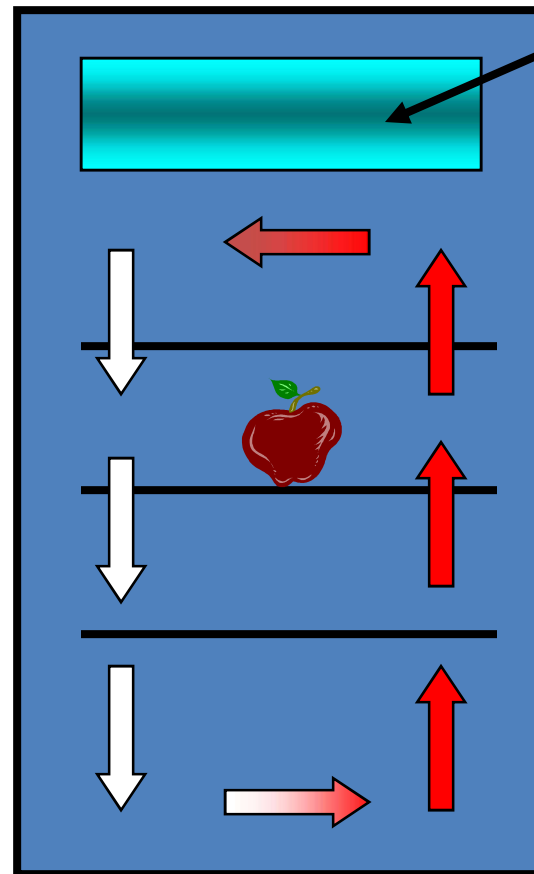
This phenomenon leads to circulation of fluid particles and is called as convection currents.



# Example of convection

Where is the freezer compartment put in a fridge?

It is put at the top, because cool air sinks, so it cools the food on the way down.



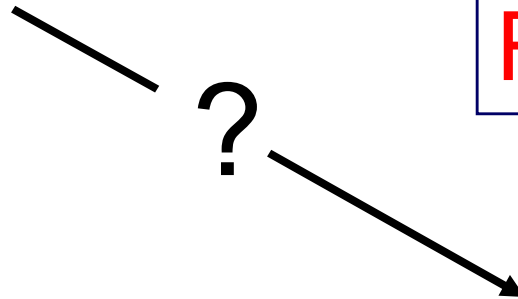
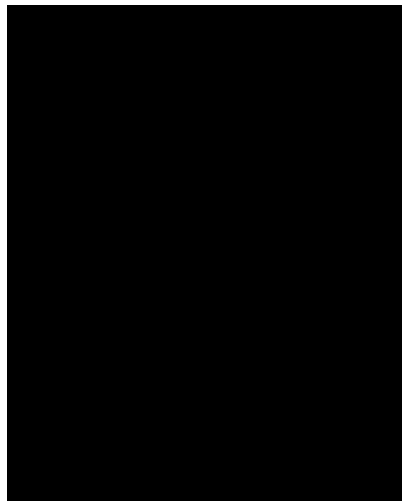
Freezer compartment

It is warmer at the bottom, so this warmer air rises and a convection current is set up.

# RADIATIONS

How does heat energy get from the Sun to the Earth?

There are no particles between the Sun and the Earth so it CANNOT travel by conduction or by convection.



**RADIATION**



# Radiation

Radiation travels in straight lines

True/~~False~~

Radiation can travel through a vacuum

True/~~False~~

Radiation requires particles to travel

~~True~~/False

Radiation travels at the speed of light

True/~~False~~

# ONE-DIMENSIONAL STEADY STATE CONDUCTION

Examples of One-dimensional Conduction:

Plate with Energy Generation and  
Variable Conductivity

## Example 2.1: Plate with internal energy generation

$q'''$  and a variable  $k$

$$k = k_o(1 - \gamma T)$$

Find temperature distribution.

### (1) Observations

- Variable  $k$
- Symmetry
- Energy generation
- Rectangular system
- Specified temperature at boundaries

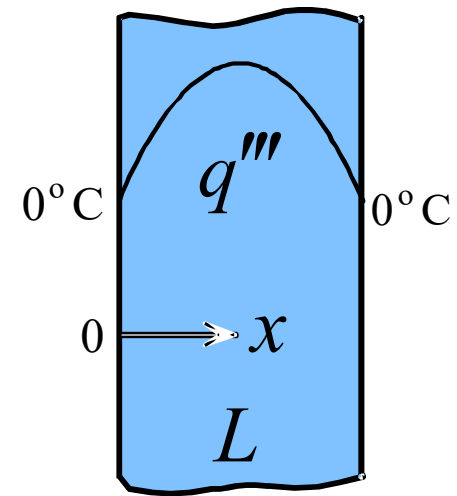


Fig. 2.1

## (2) Origin and Coordinates

Use a rectangular coordinate system

## (3) Formulation

### (i) Assumptions

- One-dimensional
- Steady
- Isotropic
- Stationary
- Uniform energy generation

## (ii) Governing Equation

Eq. (1.7):

$$\frac{d}{dx} \left( k \frac{dT}{dx} \right) + q''' = 0 \quad (2.1)$$

$$k = k_o (1 - \gamma T) \quad (a)$$

(a) into eq. (2.1)

$$\frac{d}{dx} \left[ (1 - \gamma T) \frac{dT}{dx} \right] + \frac{q'''}{k_o} = 0 \quad (b)$$

## (iii) Boundary Conditions.

Two BC are needed:

$$T(0) = 0 \quad (c)$$

$$T(L) = 0 \quad (d)$$

## (4) Solution

Integrate (b) twice

$$T + \frac{\gamma}{2} T^2 = -\frac{q'''}{2k_o} x^2 + C_1 x + C_2 \quad (e)$$

BC (c) and (d)

$$C_1 = \frac{q''' L}{2k_o} \quad , \quad C_2 = 0 \quad (f)$$



(f) into (e)

$$T^2 - \frac{2}{\gamma} T + \frac{q''' L x}{\gamma k_o} \left[ 1 - \frac{x}{L} \right] = 0 \quad (\text{g})$$

Solving for  $T$

$$T = \frac{1}{\gamma} \pm \sqrt{\frac{1}{\gamma^2} - \frac{q''' L x}{\gamma k_o} \left[ 1 - \frac{x}{L} \right]} \quad (\text{h})$$

Take the negative sign

$$T = \frac{1}{\gamma} - \sqrt{\frac{1}{\gamma^2} - \frac{q''' L x}{\gamma k_o} \left[ 1 - \frac{x}{L} \right]} \quad (\text{i})$$

## (5) Checking

- *Dimensional check*
- *Boundary conditions check*
- *Limiting check:  $q''' = 0, T = 0$*
- *Symmetry Check:*

$$\frac{dT}{dx} = -\frac{1}{2} \left[ \frac{1}{\gamma^2} - \frac{q''' L x}{\gamma k_o} \left( \frac{x}{L} - 1 \right) \right]^{-\frac{1}{2}} \left( \frac{q''' L}{\gamma k_o} \right) \left( \frac{2x}{L} - 1 \right) \quad (j)$$

Setting  $x = L/2$  in (j) gives  $dT/dx = 0$

- *Quantitative Check*

Conservation of energy and symmetry:

$$q(0) = -\frac{q'''AL}{2} \quad (\text{k})$$

$$q(L) = \frac{q'''AL}{2} \quad (\text{l})$$

Fourier's law at  $x = 0$  and  $x = L$

$$q(0) = -k_o[1 - \gamma T(0)]\frac{dT(0)}{dx} = -\frac{q'''AL}{2} \quad (\text{m})$$

$$q(L) = -k_o [1 - \gamma T(L)] \frac{dT(L)}{dx} = \frac{q''' AL}{2} \quad (\text{n})$$

## (6) Comments

Solution to the special case:

$k = \text{constant}$ : Set  $\gamma = 0$

## 2.1.2 Radial Conduction in a Composite Cylinder with Interface Friction

**Example 2.2:** Rotating shaft in sleeve, frictional heat at interface, convection on outside. Conduction in radial direction.

Determine the temperature distribution in shaft and sleeve.

### (1) Observations

- Composite cylindrical wall
- Cylindrical coordinates
- Radial conduction only

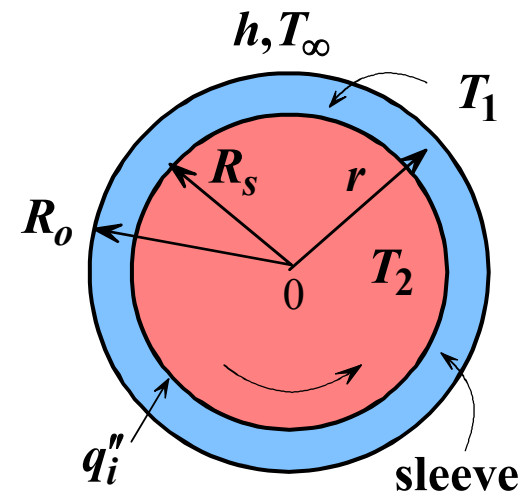


Fig. 2.2

- Steady state:

Energy generated = heat conducted through the sleeve

- No heat is conducted through the shaft
- Specified flux at inner radius of sleeve, convection at outer radius

## (2) Origin and Coordinates

Shown in Fig. 2.2

### (3) Formulation

#### (i) Assumptions

- One-dimensional radial conduction
- Steady
- Isotropic
- Constant conductivities
- No energy generation
- Perfect interface contact
- Uniform frictional energy flux
- Stationary

## (ii) Governing Equation

Shaft temperature is uniform. For sleeve: Eq. (1.11)

$$\frac{d}{dr} \left[ r \frac{dT_1}{dr} \right] = 0 \quad (2.2)$$

## (iii) Boundary Conditions

Specified flux at  $R_s$  :

$$q_i'' = -k_1 \frac{dT_1(R_s)}{dr} \quad (a)$$



Convection at  $R_o$  :

$$-k_1 \frac{dT_1(R_o)}{dr} = h[T_1(R_o) - T_\infty] \quad (b)$$

#### (4) Solution

Integrate eq. (2.2) twice

$$T_1 = C_1 \ln r + C_2 \quad (c)$$

BC give  $C_1$  and  $C_2$

$$C_1 = -\frac{q_i'' R_s}{k_1} \quad (d)$$

and

$$C_2 = T_\infty + \frac{q_i'' R_s}{k_1} \left[ \ln R_o + \frac{k_1}{hR_o} \right] \quad (e)$$

(d) and (e) into (c)

$$T_1(r) = T_\infty + \frac{q_i'' R_s}{k_1} \left[ \ln \frac{R_o}{r} + \frac{k_1}{hR_o} \right] \quad (f)$$

$hR_o / k = \text{Biot number}$

Shaft temperature  $T_2$ : Use interface boundary condition

$$T_2(r) = T_2(R_s) = T_1(R_s) \quad (g)$$

Evaluate (f) at  $r = R_s$  and use (g)

$$T_2(r) = T_\infty + \frac{q_i'' R_s}{k_1} \left[ \ln \frac{R_o}{R_s} + \frac{k_1}{h R_o} \right] \quad (h)$$

## (5) Checking

- *Dimensional check*
- *Boundary conditions check*
- *Limiting check:  $q_i'' = 0$*

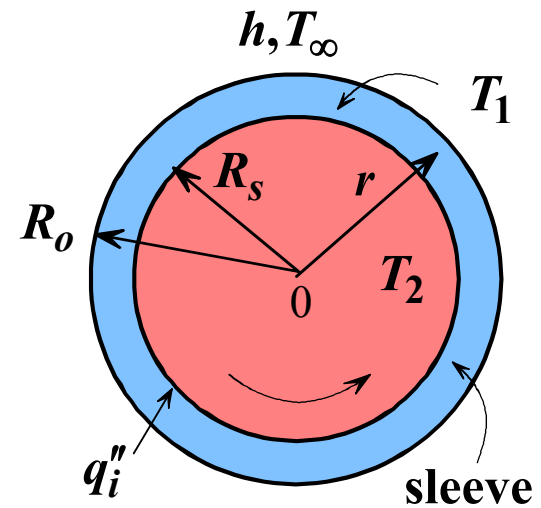


Fig. 2.2

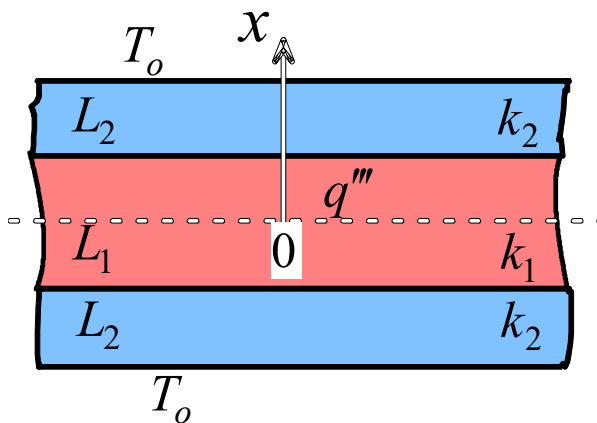
## (6) Comments

- Conductivity of shaft does not play a role

- Problem can also be treated formally as a composite cylinder. Need 2 equations and 4 BC.

## 2.1.1 Composite Wall with Energy Generation

**Example 2.1:** Plate 1 generates heat at  $q'''$ . Plate 1 is sandwiched between two plates. Outer surfaces of two plates at  $T_o$ .



**Fig. 2.3**

Find the temperature distribution in the three plates.

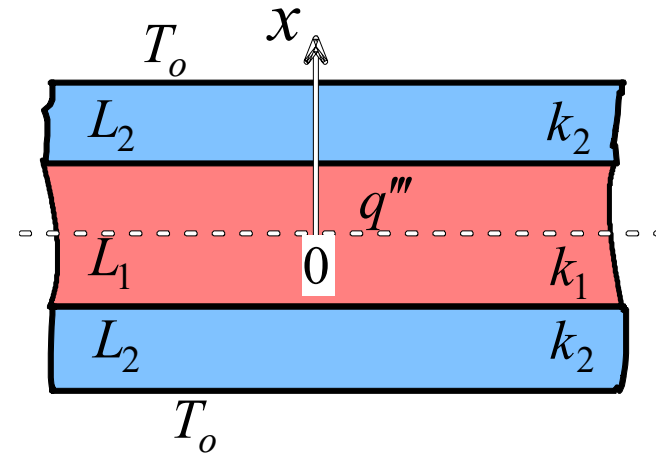
## (1) Observations

- Composite wall
- Use rectangular coordinates
- Symmetry: Insulated center plane
- Heat flows normal to plates
- Symmetry and steady state:

Energy generated = Energy conducted out

## (2) Origin and Coordinates

Shown in Fig. 2.3



**Fig. 2.3**

### (3) Formulation

#### (i) Assumptions

- Steady
- One-dimensional
- Isotropic
- Constant conductivities
- Perfect interface contact
- Stationary

#### (ii) Governing Equations

Two equations:

$$\frac{d^2 T_1}{dx^2} + \frac{q'''}{k} = 0 \quad (\text{a})$$

$$\frac{d^2 T_2}{dx^2} = 0 \quad (\text{b})$$

### (iii) Boundary Conditions

Four BC:

Symmetry:

$$\frac{dT_1(0)}{dx} = 0 \quad (\text{c})$$

Interface:

$$k_1 \frac{dT_1(L_1 / 2)}{dx} = -k_2 \frac{dT_2(L_1 / 2)}{dx} \quad (\text{d})$$

$$T_1(L_1 / 2) = T_2(L_1 / 2) \quad (\text{e})$$

Outer surface:

$$T_2(L_1 / 2 + L_2) = T_o \quad (\text{f})$$

## (4) Solution

Integrate (a) twice

$$T_1(x) = -\frac{q'''}{2k_1} x^2 + Ax + B \quad (\text{g})$$

Integrate (b)

$$T_2(x) = Cx + D \quad (\text{h})$$



Four BC give 4 constants: Solutions (g) and (h) become

$$T_1(x) = T_o + \frac{q''' L_1^2}{2k_1} \left[ \frac{1}{4} + \frac{k_1 L_2}{k_2 L_1} - \frac{x^2}{L_1^2} \right] \quad (\text{i})$$

$$T_2(x) = T_o + \frac{q''' L_1^2}{2k_2} \left[ \frac{1}{2} + \frac{L_2}{L_1} - \frac{x}{L_1} \right] \quad (\text{j})$$

## (5) Checking

- *Dimensional check*: units of  $\frac{q''' L^2}{k}$  :

$$\frac{q'''(\text{W/m}^3)L^2(\text{m}^2)}{k(\text{W/m-}^\circ\text{C})} = ^\circ\text{C}$$

- *Boundary conditions check*
- *Quantitative check:*

1/2 the energy generated in center plate = Heat  
conducted at  $x = L_1 / 2$

$$\frac{L_1}{2} q''' = -k_1 \frac{dT_1(L_1 / 2)}{dx} \quad (\text{k})$$

(i) into (k)

$$-k_1 \frac{dT_1(L_1 / 2)}{dx} = \frac{L_1}{2} q'''$$

Similarly, 1/2 the energy generated in center plate  
= Heat conducted out

$$\frac{L_1}{2} q''' = -k_2 \frac{dT_2(L_1 / 2 + L_2)}{dx} \quad (1)$$

(j) into (1) shows that this condition is satisfied.

- *Limiting check:*

(i) If  $q''' = 0$ , then  $T_1(x) = T_2(x) = T_o$ .

(ii) If  $L_1 = 0$  then  $T_1(x) = T_o$ .

## (6) Comments

Alternate approach: Outer plate with a specified flux at  $x = L_1 / 2$  and a specified temperature at  $x = L_1 / 2 + L_2$ .

## 2.2 Extended Surfaces - Fins

### 2.2.1 The Function of Fins

Newton's law of cooling:

$$q_s = hA_s(T_s - T_\infty) \quad (2.3)$$

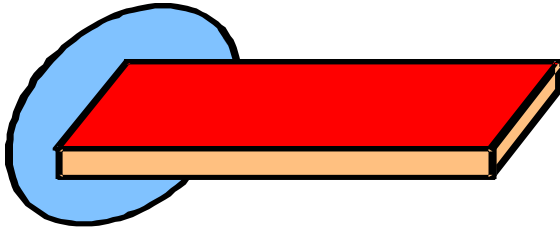
Options for increasing  $q_s$  :

- Increase  $h$
- Lower  $T_\infty$
- Increase  $A_s$

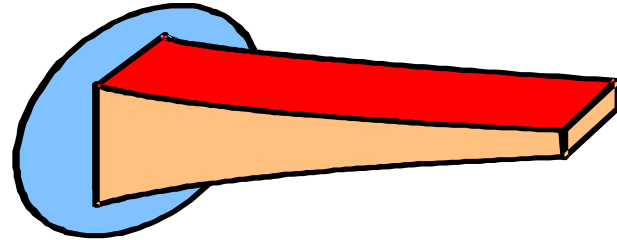
Examples of Extended Surfaces (Fins):

- Thin rods on condenser in back of refrigerator
- Honeycomb surface of a car radiator
- Corrugated surface of a motorcycle engine
- Disks or plates used in baseboard radiators

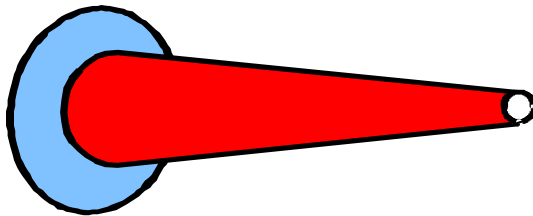
## 2.2.2 Types of Fins



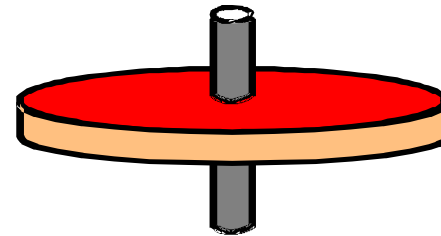
(a) constant area  
straight fin



(b) variable area  
straight fin



(c) pin fin



(d) annular fin

Fig. 2.5

## Terminology and types

- Fin base
- Fin tip
- Straight fin
- Variable cross-sectional area fin
- Spine or pin fin
- Annular or cylindrical fin

## 2.2.3 Heat Transfer and Temperature Distribution in Fins

- Heat flows axially and laterally (two-dimensional)
- Temperature distribution is two-dimensional

## 2.2.4 The Fin Approximation

Neglect lateral temperature variation

$$T \approx T(x)$$

Criterion:

Biot number =  $Bi$

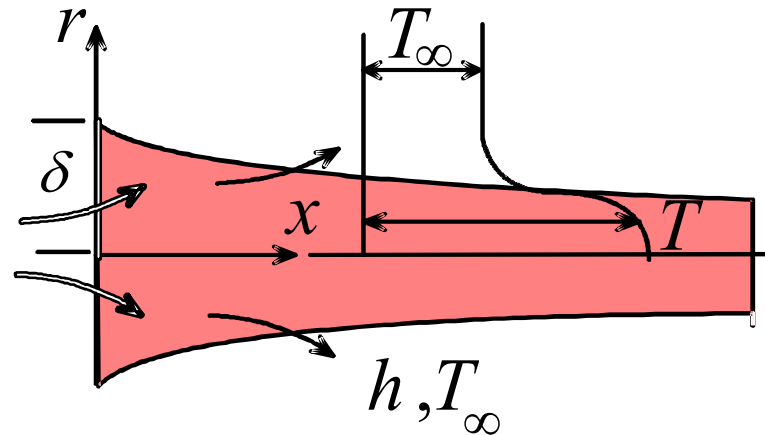


Fig. 2.6

$$Bi = h\delta/k \ll 1 \quad (2.4)$$

$$Bi = \frac{\delta / k}{1 / h} = \frac{\text{Internal resistance}}{\text{external resistance}}$$



## 2.2.5 The Fin Heat Equation: Convection at Surface

### (1) Objective:

Determine fin heat transfer rate.

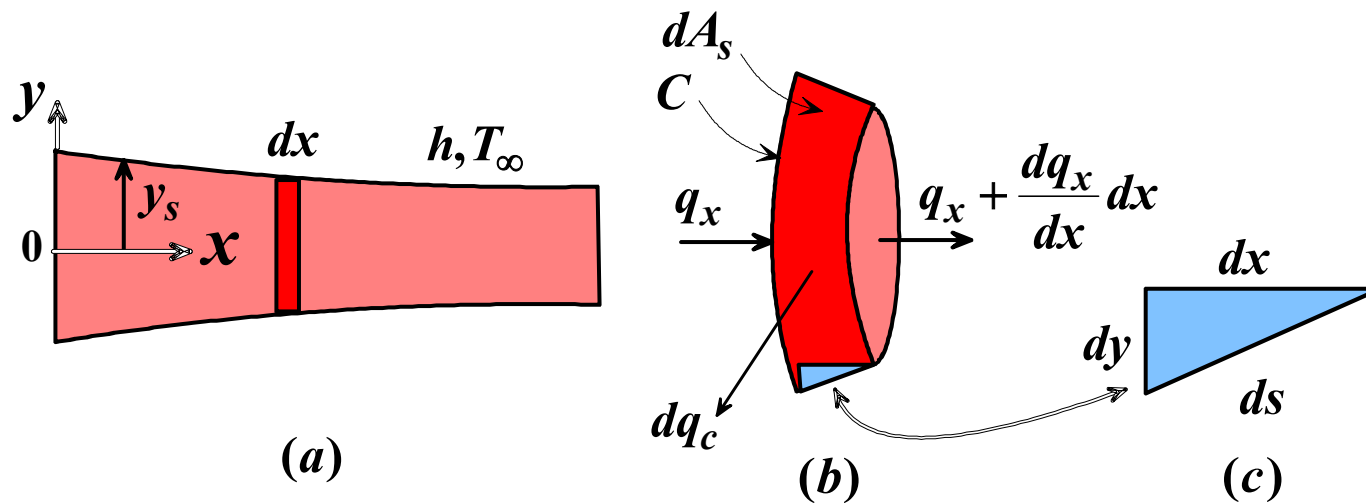
Need temperature distribution.

### (2) Procedure:

Formulate the fin heat equation.

Apply conservation of energy.

- Select an origin and coordinate axis  $x$ .
- Assume  $Bi < 0.1$ ,  $\therefore T = T(x)$
- Stationary material, steady state



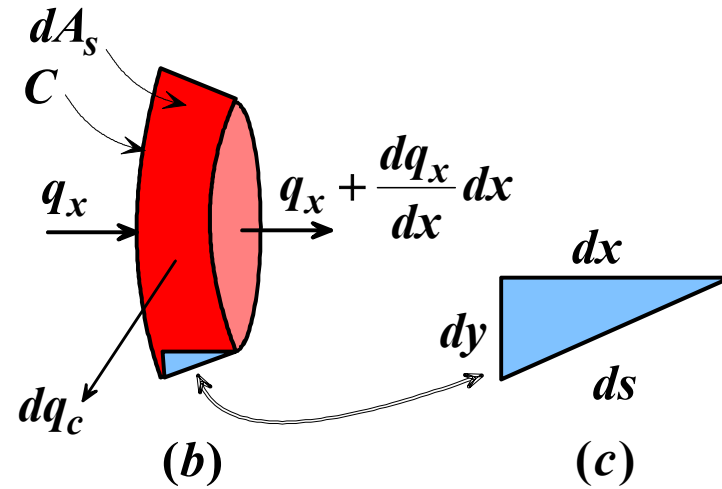
**Fig. 2.7**

Conservation of energy for the element  $dx$ :

$$\dot{E}_{in} + \dot{E}_g = \dot{E}_{out} \quad (a)$$

$$\dot{E}_{in} = q_x \quad (b)$$

$$\dot{E}_{out} = q_x + \frac{dq_x}{dx} dx + dq_c \quad (c)$$



(b) and (c) into (a)

$$\dot{E}_g = \frac{dq_x}{dx} dx + dq_c \quad (d)$$

Fourier's law and Newton's law

$$q_x = -kA_c \frac{dT}{dx} \quad (e)$$

$$dq_c = h(T - T_\infty) dA_s \quad (f)$$

Energy generation

$$\dot{E}_g = q''' A_c(x) dx \quad (g)$$

(e), (f) and (g) into (d)

$$\frac{d}{dx} \left[ k A_c(x) \frac{dT}{dx} \right] dx - h (T - T_\infty) dA_s + q''' A_c(x) dx = 0 \quad (2.5a)$$

Assume constant  $k$

$$\frac{d^2 T}{dx^2} + \frac{1}{A_c(x)} \frac{dA_c}{dx} \frac{dT}{dx} - \frac{h}{k A_c(x)} (T - T_\infty) \frac{dA_s}{dx} + \frac{q'''}{k} = 0 \quad (2.5b)$$

- (2.5b) is the heat equation for fins
- **Assumptions:**
  - (1) Steady state
  - (2) Stationary

- (3) Isotropic
- (4) Constant  $k$
- (5) No radiation
- (6)  $Bi \ll 1$
- $A_c$ ,  $dA_c / dx$ , and  $dA_s / dx$  are determined from the geometry of fin.

## 2.2.6 Determination of $dA_s / dx$

From Fig. 2.7b

$$dA_s = C(x) ds \quad (a)$$

$C(x)$  = circumference

$ds$  = slanted length of the element

For a right triangle

$$ds = [dx^2 + dy_s^2]^{1/2} \quad (b)$$

(b) into (a)

$$\frac{dA_s}{dx} = C(x) \left[ 1 + \left( \frac{dy_s}{dx} \right)^2 \right]^{1/2} \quad (2.6a)$$

For  $dy_s / dx \ll 1$

$$\frac{dA_s}{dx} = C(x) \quad (2.6b)$$

## 2.2.7 Boundary Conditions

Need two BC

## 2.2.8 Determination of Fin Heat Transfer

Rate  $q_f$  :

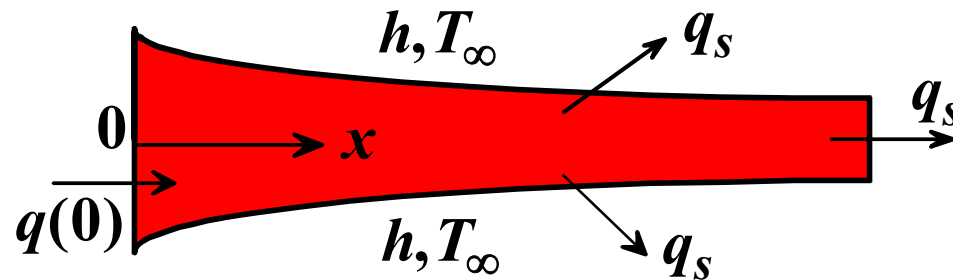


Fig. 2.8

Conservation of energy for  $q''' = 0$  :

$$q_f = q(0) = q_s \quad (a)$$

Two methods to determine  $q_f$  :

(1) *Conduction at base.*

Fourier's law at  $x = 0$

$$\mathbf{q}_f = \mathbf{q}(0) = -kA_c(0) \frac{dT(0)}{dx} \quad (2.7)$$

(2) *Convection at the fin surface.*

Newton's law applied at the fin surface

$$\mathbf{q}_f = \mathbf{q}_s = \int_{A_s} h[T(x) - T_\infty] dA_s \quad (2.8)$$

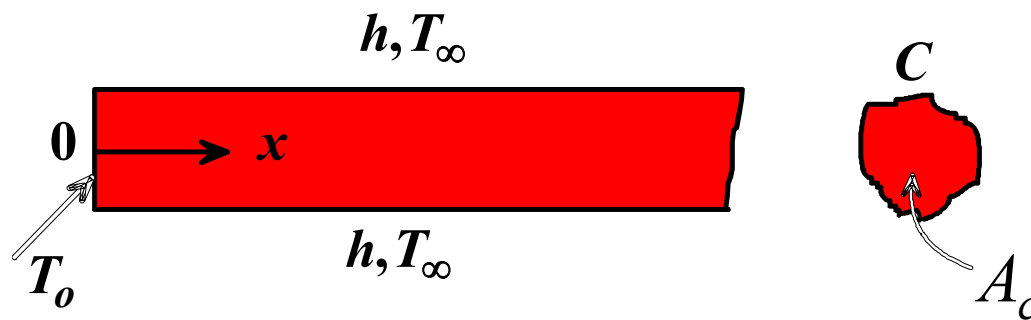
- Fin attached at both ends: Modify eq. (2.7) accordingly

- Fin with convection at the tip: Integral in eq. (2.8) includes tip



- Convection and radiation at surface: Apply eq. (2.7).  
Modify eq. (2.8) to include heat exchange by radiation.

## 2.2.9 Applications: Constant Area Fins with Surface Convection



**Fig. 2.9**

## A. Governing Equation

Use eq. (2.5b). Set

$$dA_c / dx = 0 \quad (a)$$

$$y_s = \text{constant}$$

$$dy_s / dx = 0$$

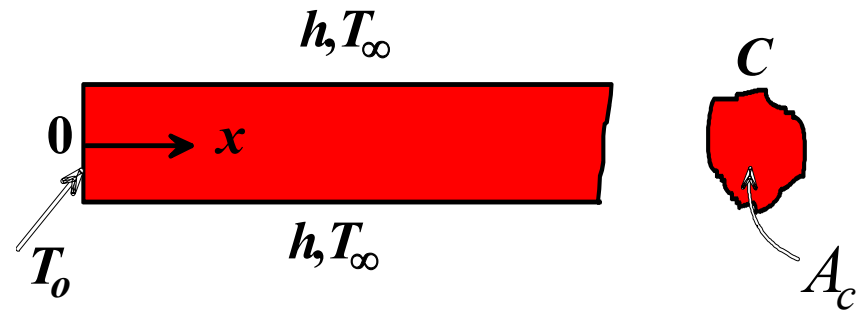


Fig. 2.9

Eq. (2.6a)

$$dA_s / dx = C \quad (b)$$

(a) and (b) into eq. (2.5b)

$$\frac{d^2 T}{dx^2} - \frac{hC}{kA_c} (T - T_\infty) = 0 \quad (2.9)$$

Rewrite eq. (2.9)

$$\theta = T - T_{\infty} \quad (c)$$

$$m^2 = \frac{hC}{kA_c} \quad (d)$$

Assume  $T_{\infty} = \text{constant}$ , (c) and (d) into (2.9)

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \quad (2.10)$$

Valid for:

(1) Steady state

(2) constant  $k$ ,  $A_c$  and  $T_{\infty}$

(3) No energy generation

(4) No radiation

(5)  $Bi \ll 1$

(6) Stationary fin

## B. Solution

Assume:  $h = \text{constant}$

$$\theta(x) = A_1 \exp(mx) + A_2 \exp(-mx) \quad (2.11a)$$

$$\theta(x) = B_1 \sinh mx + B_2 \cosh mx \quad (2.11b)$$

## C. Special Case (i):

- Finite length
- Specified temperature at base, convection at tip

Boundary conditions:

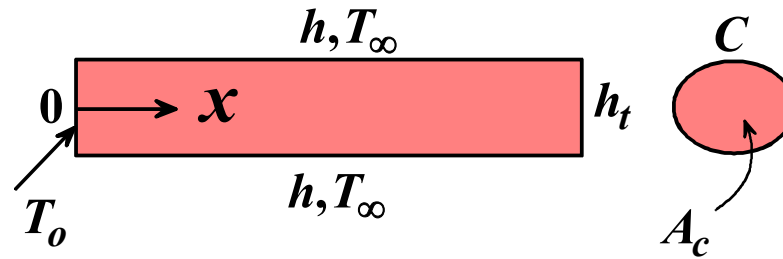


Fig. 2.10

$$T(0) = T_0 \quad (e)$$

$$-k \frac{dT(L)}{dx} = h_t [T(L) - T_\infty] \quad (f)$$

$$\theta(0) = \theta_0 \quad (h)$$

$$-k \frac{d\theta(L)}{dx} = h_t \theta(L) \quad (i)$$

Two BC give  $B_1$  and  $B_2$

$$\begin{aligned} \frac{\theta(x)}{\theta_o} &= \frac{T(x) - T_\infty}{T_o - T_\infty} \\ &= \frac{\cosh m(L-x) + (h_t/mk) \sinh m(L-x)}{\cosh mL + (h_t/mk) \sinh mL} \end{aligned} \quad (2.12)$$

Eq. (2.7) gives  $q_f$

$$q_f = [k A_c C h]^{1/2} \frac{(T_o - T_\infty) [\sinh mL + (h_t/mk) \cosh mL]}{\cosh mL + (h_t/mk) \sinh mL} \quad (2.13)$$

## C. Special Case (ii):

- Finite length
- Specified temperature at base, insulated tip

BC at tip:

$$\frac{d\theta(L)}{dx} = 0 \quad (j)$$

Set  $h_t = 0$  eq. (2.12)

$$\frac{\theta(x)}{\theta_o} = \frac{T(x) - T_\infty}{T_o - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL} \quad (2.14)$$

Set  $h_t = 0$  eq. (2.13)

$$q_f = [kA_c Ch]^{1/2} (T_o - T_\infty) \tanh mL \quad (2.15)$$

## 2.2.10 Corrected Length $L_c$

- Insulated tip: simpler solution
- Simplified model: Assume insulated tip, compensate by increasing length by  $\Delta L_c$

- The *corrected length* is  $L_c$

$$L_c = L + \Delta L_c \quad (2.16)$$

- The *correction increment*  $\Delta L_c$  *depends on the geometry of the fin:*

Increase in surface area due to  $\Delta L_c =$  tip area

Circular fin:

$$\pi r_0^2 = 2\pi r_0 \Delta L_c$$



$$\Delta L_c = r_o / 2$$

Square bar of side  $t$

$$\Delta L_c = t / 4$$

### 2.2.11 Fin Efficiency $\eta_f$

Definition

$$\eta_f = \frac{q_f}{q_{\max}} \quad (2.17)$$

$$q_{\max} = hA_s(T_o - T_{\infty})$$

$A_s$  = total surface area

Eq. (2.17) becomes

$$\eta_f = \frac{q_f}{hA_s(T_o - T_\infty)} \quad (2.18)$$

# **Radiation Heat Transfer**



# Blackbody Radiation

- Blackbody – a perfect emitter & absorber of radiation
- Emits radiation uniformly in all directions – no directional distribution – it's diffuse
- Joseph Stefan (1879)– total radiation emission per unit time & area over all wavelengths and in all directions:

$$E_b = \sigma T^4 \left( \text{W/m}^2 \right)$$

- $\sigma$ =Stefan-Boltzmann constant =  $5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

# Planck's Distribution Law

- Sometimes we're interested in radiation at a certain wavelength
- Spectral blackbody emissive power ( $E_{b\lambda}$ ) = “amount of radiation energy emitted by a blackbody at an absolute temperature  $T$  per unit time, per unit surface area, and per unit wavelength about the wavelength  $\lambda$ .”

# Planck's Distribution Law

- For a surface in a vacuum or gas

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} \quad (\text{W/m}^2 \cdot \mu\text{m})$$

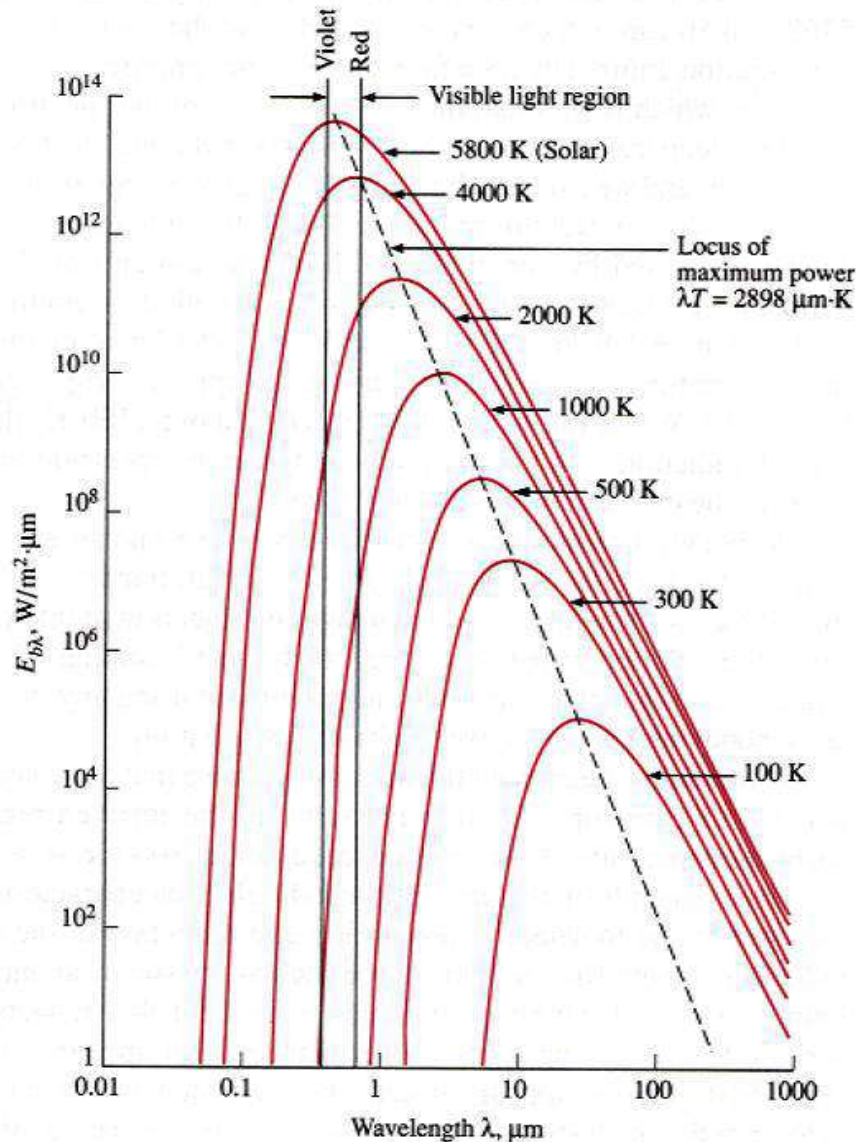
where

$$C_1 = 2\pi h c_o^2 = 3.742 \times 10^8 \text{ W} \cdot \mu\text{m}^4 / \text{m}^2$$

$$C_2 = h c_o / k = 1.439 \times 10^4 \mu\text{m} \cdot \text{K}$$

$$k = 1.3805 \times 10^{-23} \text{ J/K} = \text{Boltzmann's constant}$$

- Other media: replace  $C_1$  with  $C_1/n^2$
- Integrating this function over all  $\lambda$  gives us the equation for  $E_b$ .



# Radiation Distribution

- Radiation is a continuous function of wavelength
- Radiation increases with temp.
- At higher temps, more radiation is at shorter wavelengths.
- Solar radiation peak is in the visible range.

# Wien's Displacement Law

- Peak can be found for different temps using Wien's Displacement Law:

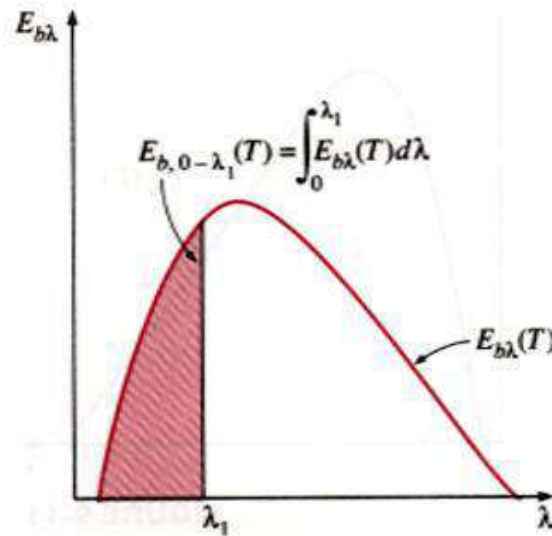
$$(\lambda T)_{max\ power} = 2897.5\ \mu\text{m} \cdot \text{K}$$

- Note that color is a function of absorption & reflection, not emission



# Blackbody Radiation Function

- We often are interested in radiation energy emitted over a certain wavelength.



- This is a tough integral to do!

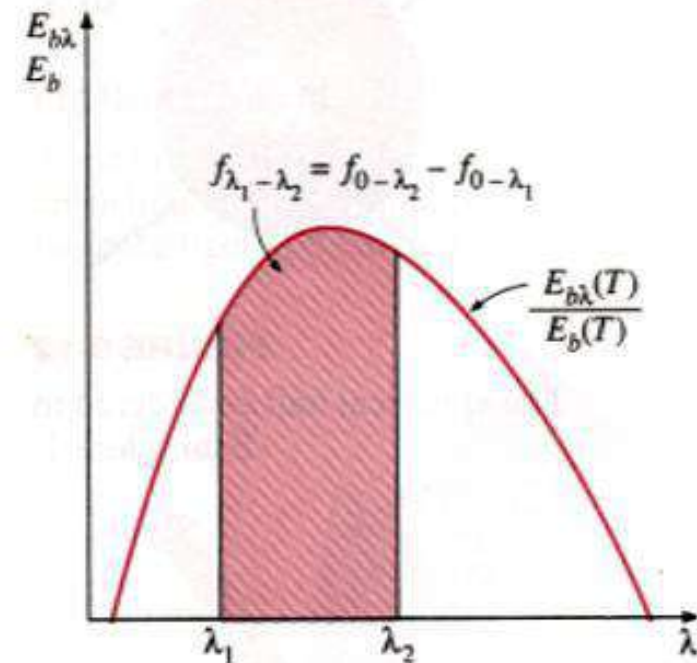
# Blackbody Radiation Function

- Use blackbody radiation function,  $F_\lambda$

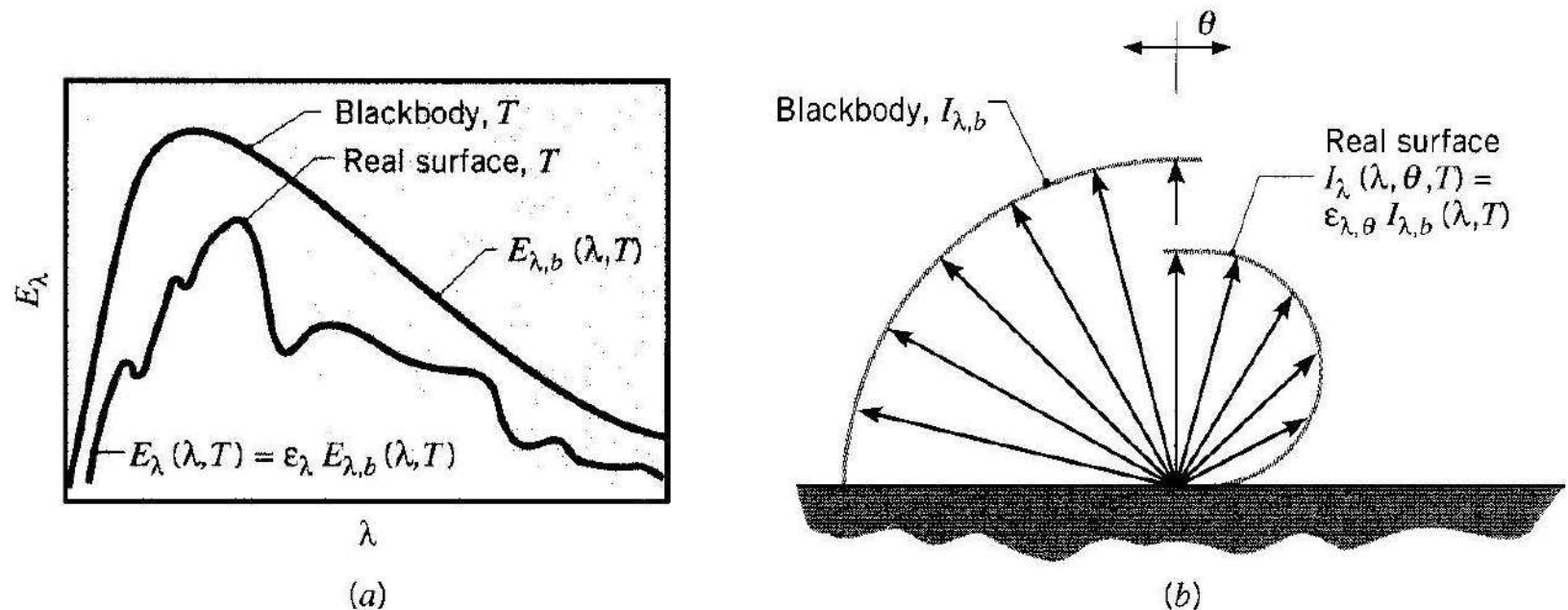
$$F_\lambda(T) = \frac{\int_0^\lambda E_{b\lambda}(T) d\lambda}{\sigma T^4}$$

- If we want radiation between  $\lambda_1$  &  $\lambda_2$ ,

$$F_{\lambda_1-\lambda_2}(T) = F_{\lambda_2}(T) - F_{\lambda_1}(T)$$



# Surface Emission



**FIGURE 12.16** Comparison of blackbody and real surface emission. (a) Spectral distribution. (b) Directional distribution.

# Surface Emission

Spectral, directional emissivity

$$\epsilon_{\lambda,\theta}(\lambda, \theta, \phi, T) \equiv \frac{I_{\lambda,e}(\lambda, \theta, \phi, T)}{I_{\lambda,b}(\lambda, T)}$$

Total, directional emissivity

$$\epsilon_{\theta}(\theta, \phi, T) \equiv \frac{I_e(\theta, \phi, T)}{I_b(T)}$$

Spectral, hemispherical emissivity

$$\epsilon_{\lambda}(\lambda, T) \equiv \frac{E_{\lambda}(\lambda, T)}{E_{\lambda,b}(\lambda, T)}$$

$$\epsilon_{\lambda}(\lambda, T) = \frac{\int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi, T) \cos \theta \sin \theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,b}(\lambda, T) \cos \theta \sin \theta \, d\theta \, d\phi}$$

# Surface Emission

Spectral, hemispherical emissivity

$$\varepsilon_{\lambda}(\lambda, T) \equiv \frac{E_{\lambda}(\lambda, T)}{E_{\lambda,b}(\lambda, T)}$$

Substituting spectral emissive power

$$\varepsilon_{\lambda}(\lambda, T) = \frac{\int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi, T) \cos \theta \sin \theta d\theta d\phi}{\int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,b}(\lambda, T) \cos \theta \sin \theta d\theta d\phi}$$

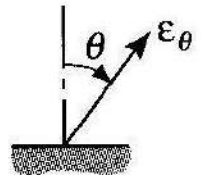
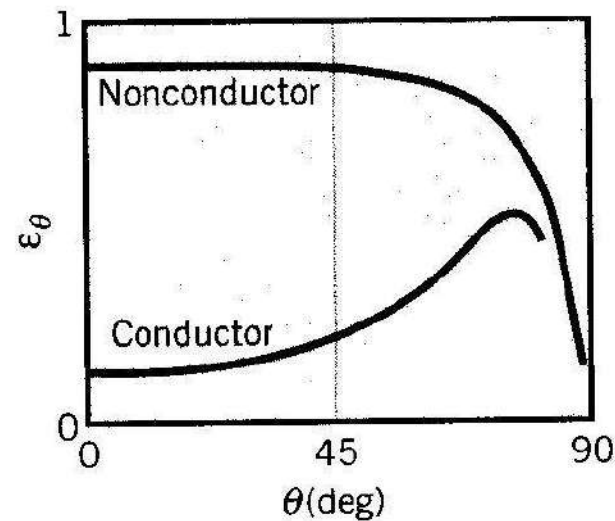
# Surface Emission

Total, hemispherical emissivity

$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)}$$

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{\lambda,b}(\lambda, T) d\lambda}{E_b(T)}$$

Total, directional emissivity  
Normal emissivity predictable





# Spectral, normal emissivity

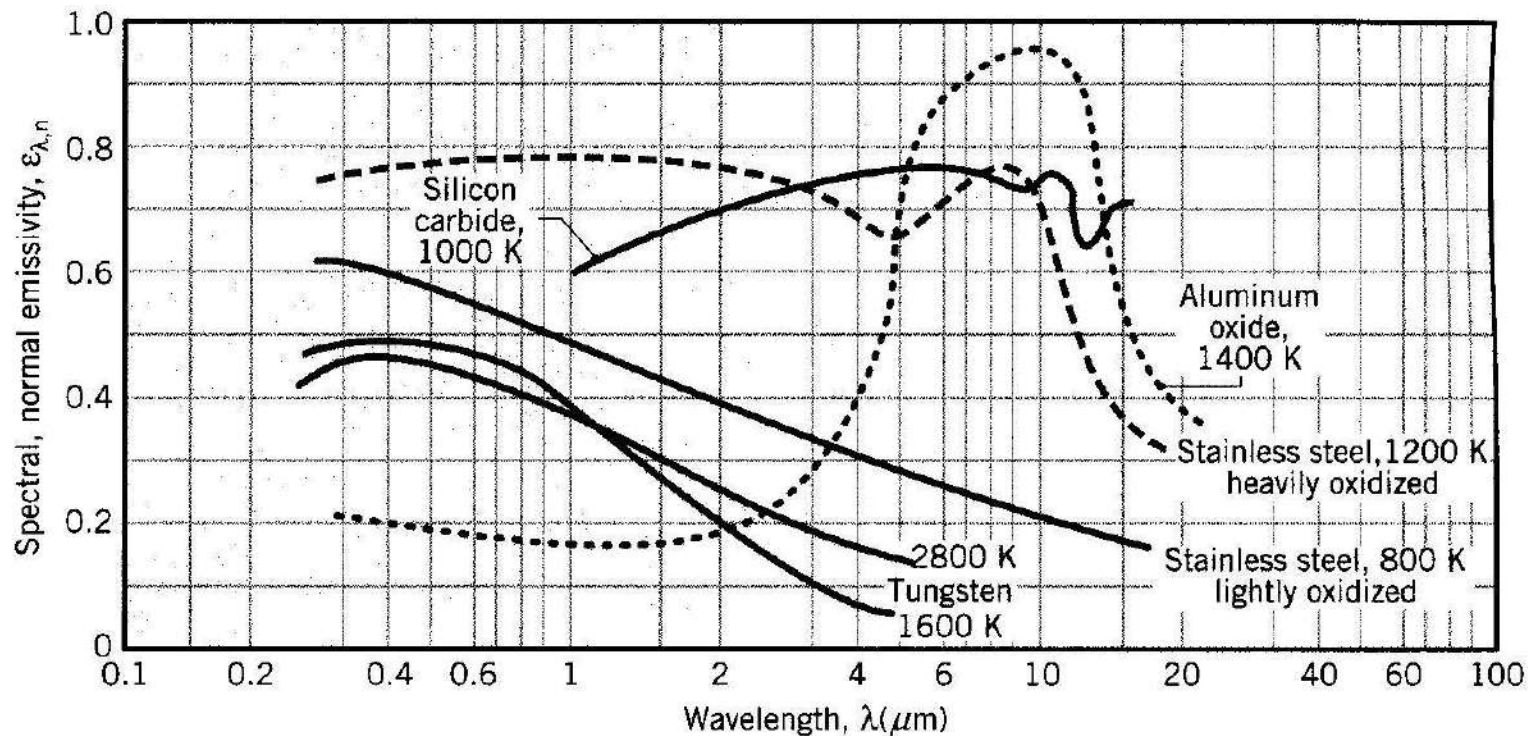
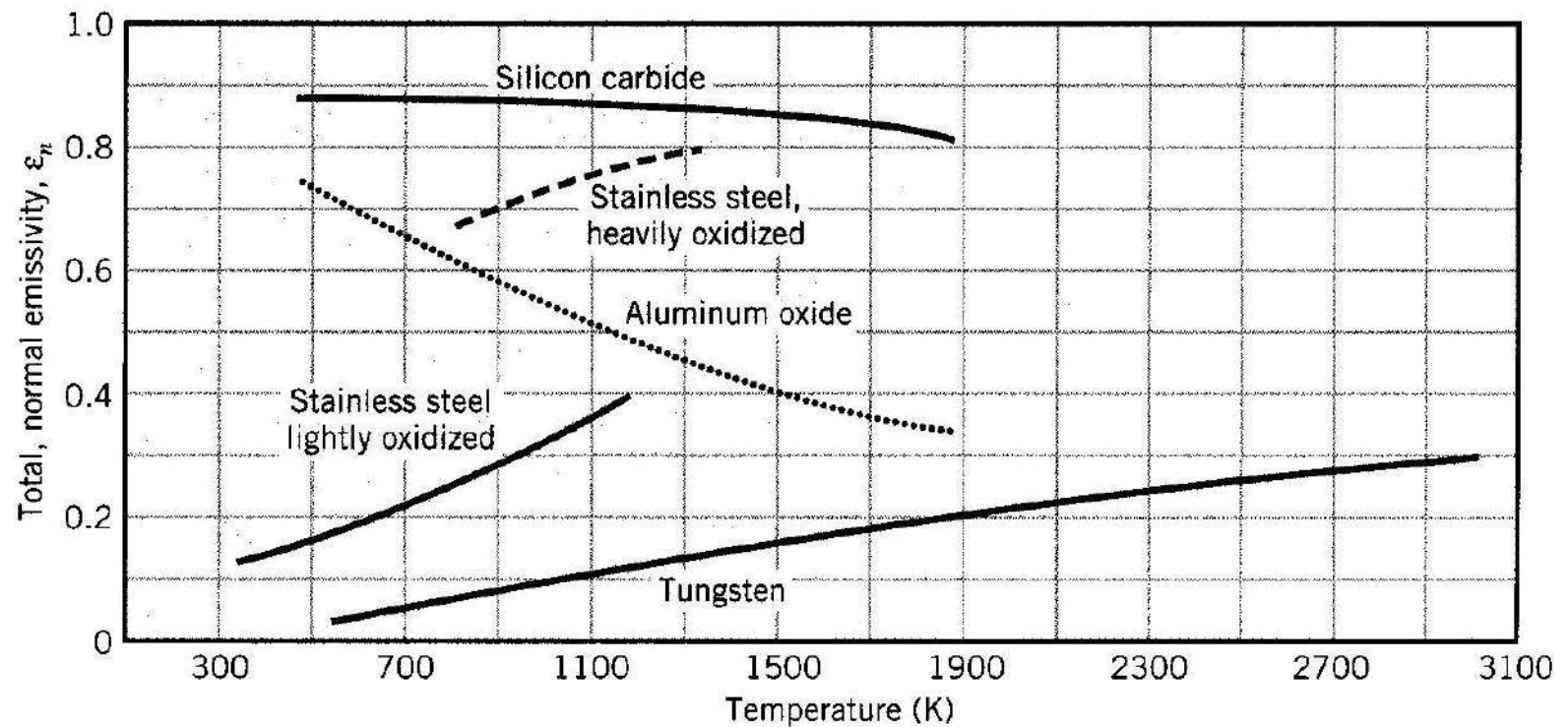


FIGURE 12.18 Spectral dependence of the spectral, normal emissivity  $\epsilon_{\lambda,n}$  of selected materials.

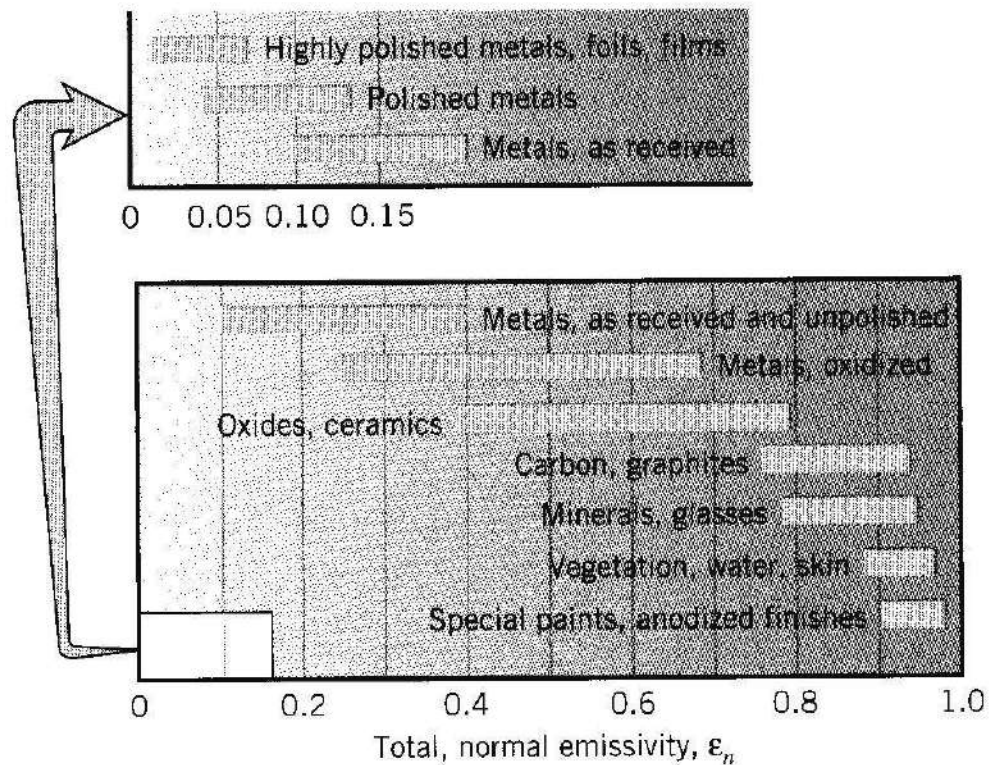
# Total, normal emissivity



**FIGURE 12.19** Temperature dependence of the total, normal emissivity  $\epsilon_n$  of selected materials.



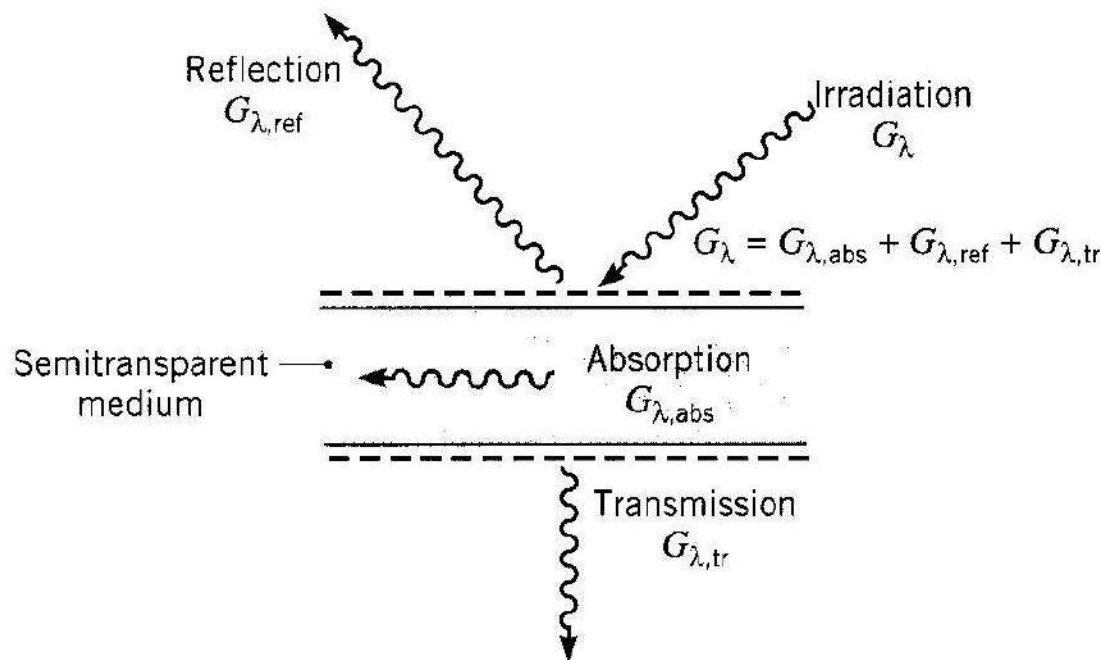
# Total, normal emissivity



**FIGURE 12.20** Representative values of the total, normal emissivity  $\epsilon_n$ .

# Absorption, Reflection, and Transmission

$$G_{\lambda} = G_{\lambda,\text{ref}} + G_{\lambda,\text{abs}} + G_{\lambda,\text{tr}}$$



**FIGURE 12.21**

Absorption, reflection, and transmission processes associated with a semitransparent medium.

# Absorptivity

Spectral, directional absorptivity

$$\alpha_{\lambda,\theta}(\lambda, \theta, \phi) \equiv \frac{I_{\lambda,i,\text{abs}}(\lambda, \theta, \phi)}{I_{\lambda,i}(\lambda, \theta, \phi)}$$

Spectral, hemispherical absorptivity

$$\alpha_{\lambda}(\lambda) \equiv \frac{G_{\lambda,\text{abs}}(\lambda)}{G_{\lambda}(\lambda)}$$

Total, hemispherical absorptivity

$$\alpha \equiv \frac{G_{\text{abs}}}{G}$$

# Reflectivity

Spectral, directional reflectivity

$$\rho_{\lambda,\theta}(\lambda, \theta, \phi) \equiv \frac{I_{\lambda,i,\text{ref}}(\lambda, \theta, \phi)}{I_{\lambda,i}(\lambda, \theta, \phi)}$$

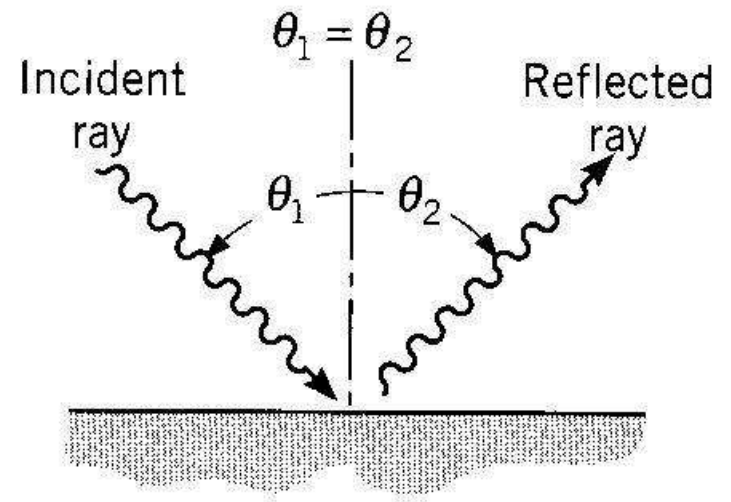
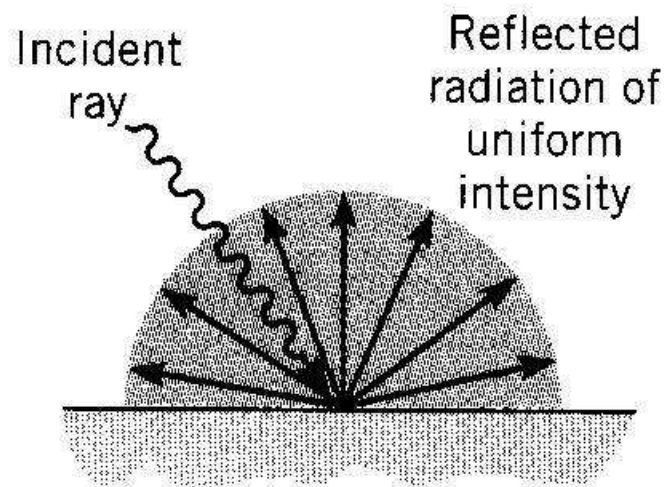
Spectral, hemispherical reflectivity

$$\rho_{\lambda}(\lambda) \equiv \frac{G_{\lambda,\text{ref}}(\lambda)}{G_{\lambda}(\lambda)}$$

Total, hemispherical reflectivity

$$\rho \equiv \frac{G_{\text{ref}}}{G}$$

# Reflectivity



**FIGURE 12.22** Diffuse and specular reflection.



# Transmissivity

Spectral, hemispherical transmissivity

$$\tau_{\lambda} = \frac{G_{\lambda, \text{tr}}(\lambda)}{G_{\lambda}(\lambda)}$$

Total, hemispherical reflectivity

$$\tau = \frac{G_{\text{tr}}}{G}$$

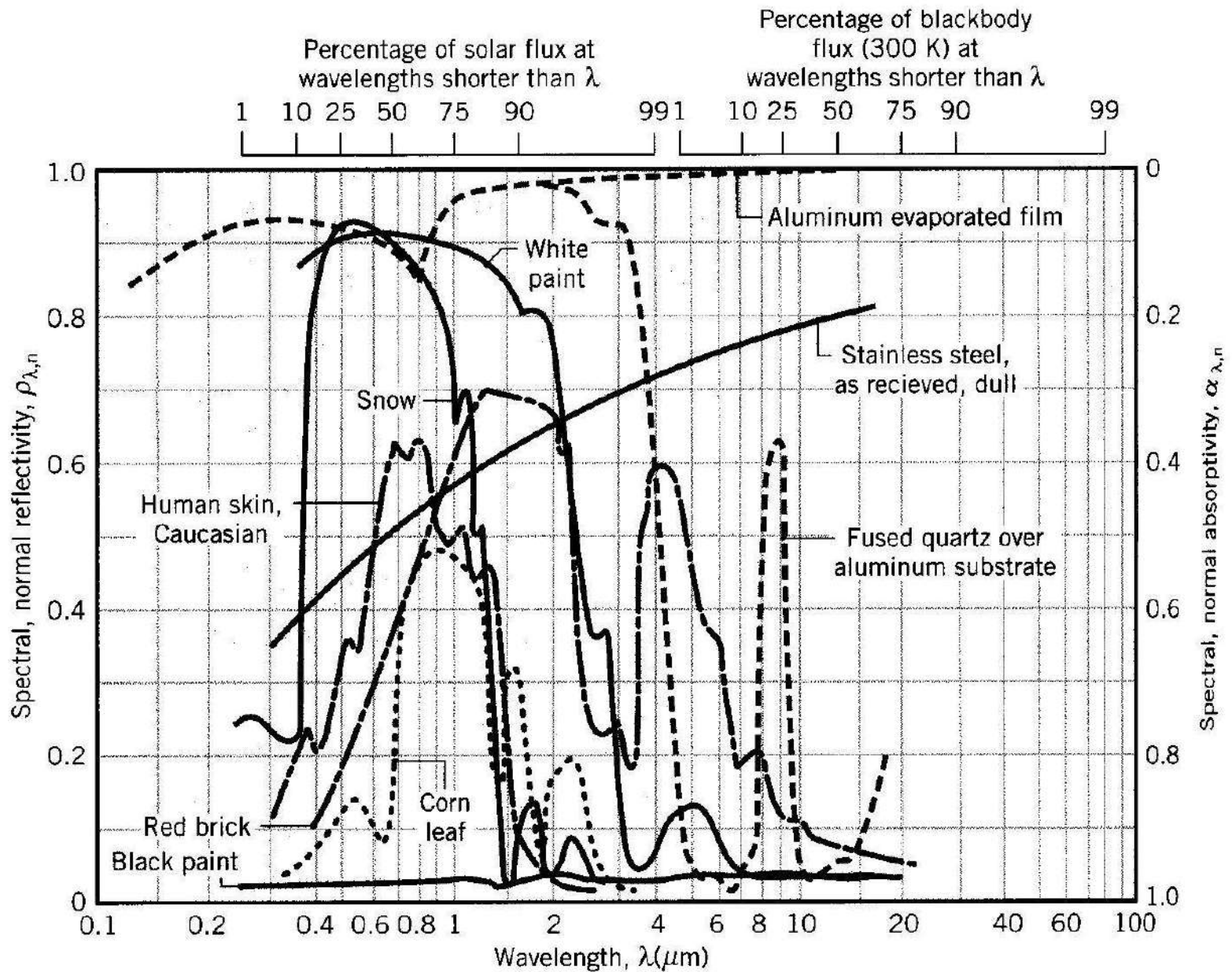
# Special Considerations

Semitransparent medium

$$\rho + \alpha + \tau = 1$$

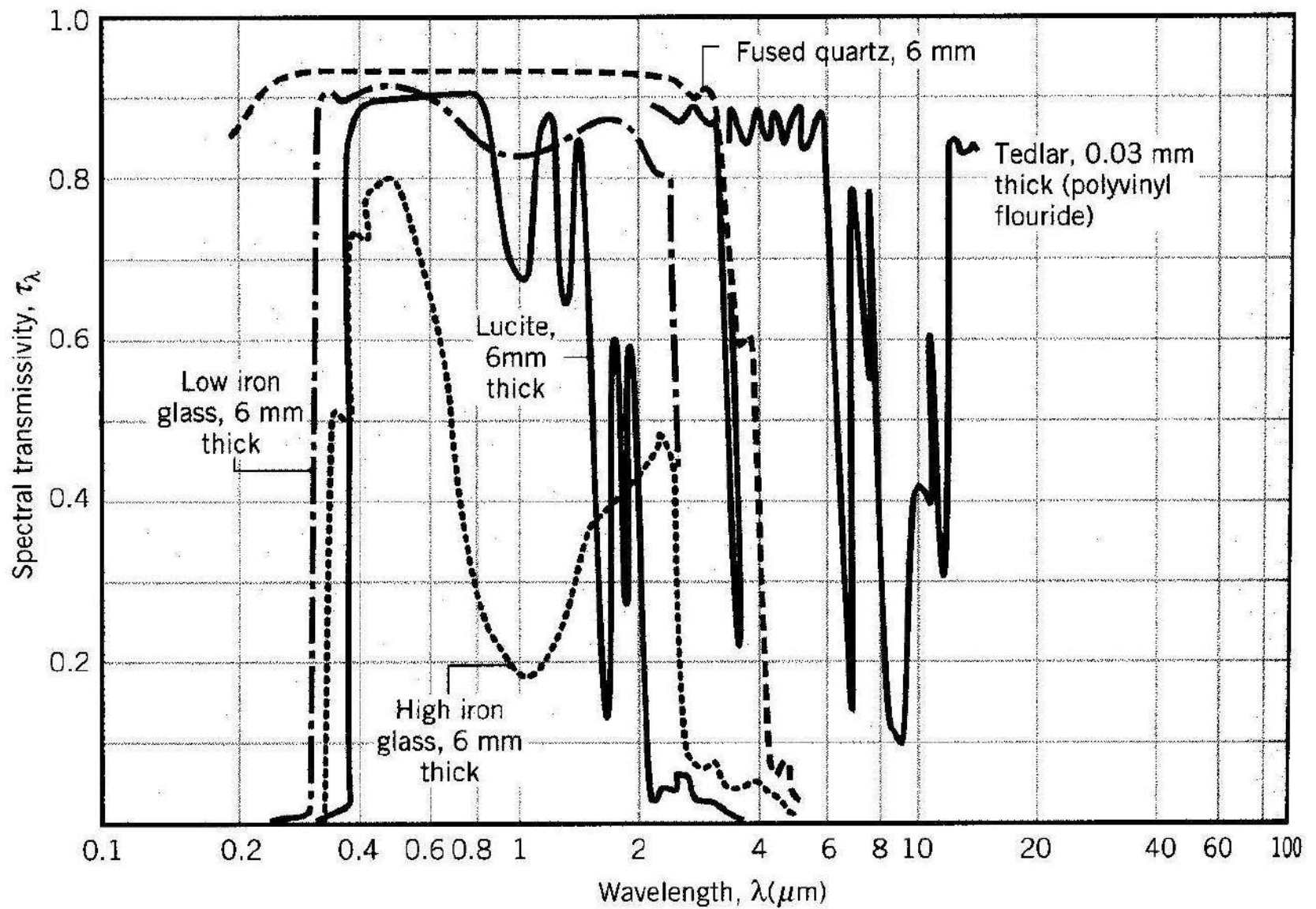
Opaque

$$\rho + \alpha = 1$$



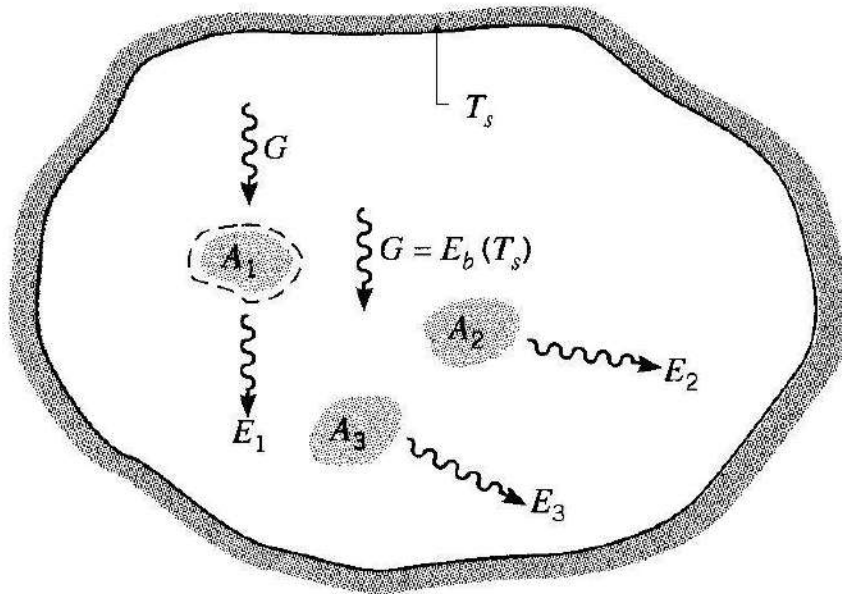
**FIGURE 12.23** Spectral dependence of the spectral, normal absorptivity  $\alpha_{\lambda,n}$  and reflectivity  $\rho_{\lambda,n}$  of selected opaque materials.





**FIGURE 12.24** Spectral dependence of the spectral transmissivities  $\tau_\lambda$  of selected semitransparent materials.

# Kirchhoff's Law



$$\varepsilon = \alpha$$
$$\frac{E_1(T_s)}{\alpha_1} = E_b(T_s)$$

**FIGURE 12.25**  
Radiative exchange in an isothermal enclosure.

No real surface can have an emissive power exceeding that of a black surface at the same temperature.