Electromagnetic Waves Maxwell`s Equations, Transmission Lines and Waveguides

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The Electromagnetic Spectrum

We give different names to different "parts" of the electromagnetic spectrum. These "parts" are separated according to wavelength.

These names are very familiar to you.

Your eyes are sensitive to only the *very tiny* part of the spectrum which we call "visible light".

Your eyes are most sensitive to green and yellow light. Your eyes are not very sensitive to red and blue light.

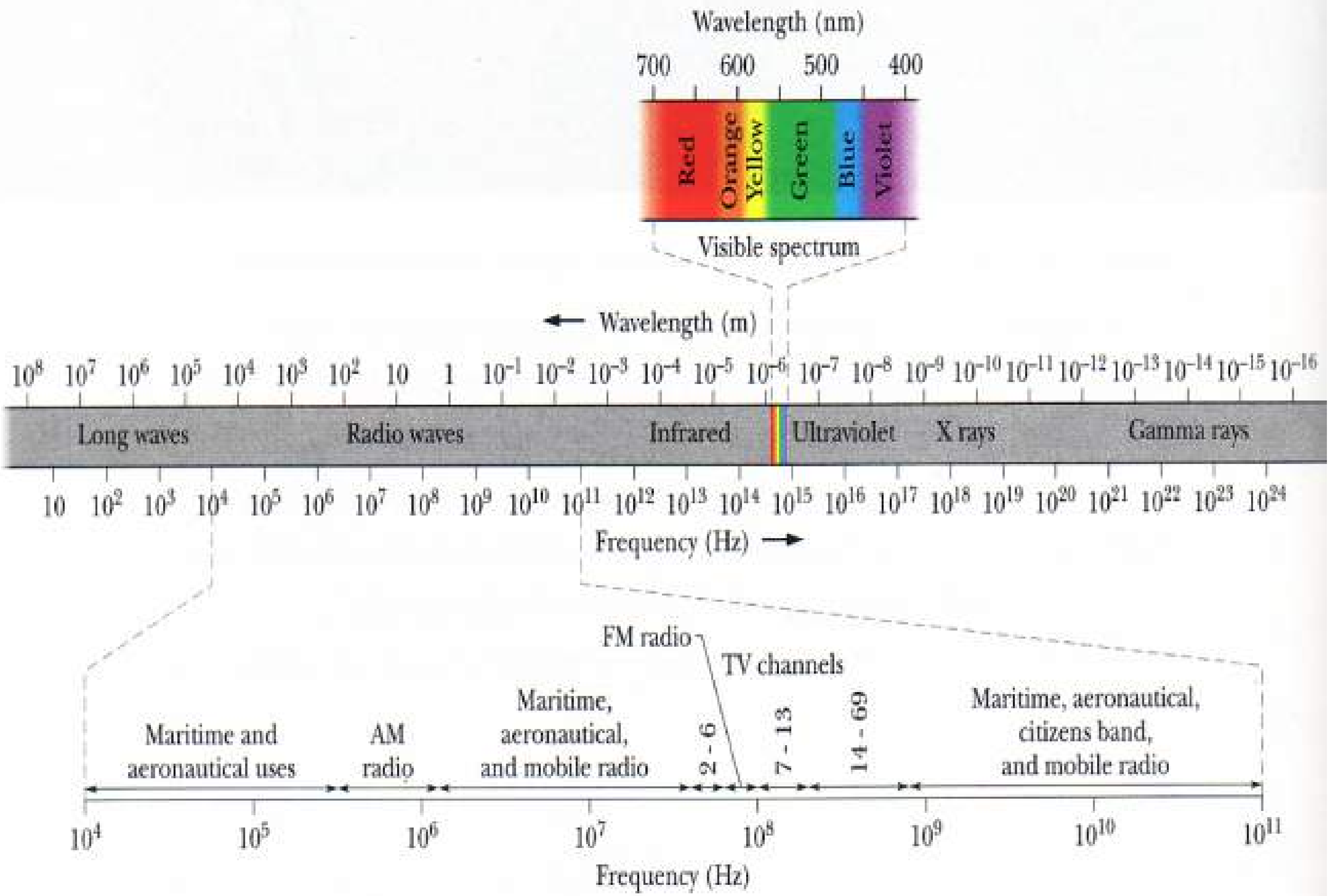


FIGURE 34-1 The electromagnetic spectrum.

Electromagnetic Waves

A changing electric field gives rise to a
changing magnetic field, which gives rise to a
changing electric field, which gives rise to a
changing magnetic field, which ...

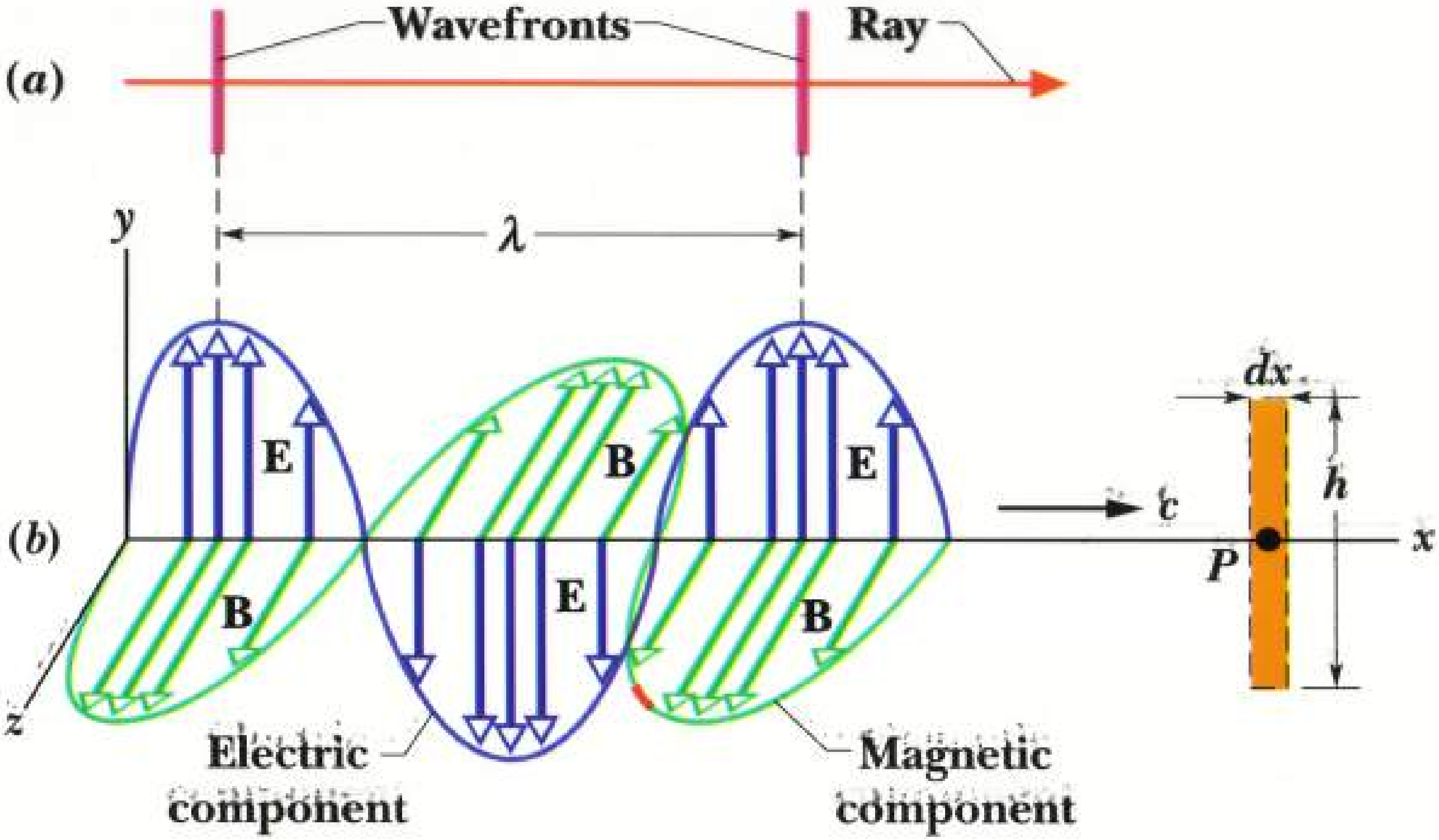
$\Rightarrow$ You don't need any charges or currents around to produce "waving" E and B fields.

All electromagnetic waves move at the speed

$$c = 299792458 \text{ m/sec}$$

The "speed of light"

$$c = \lambda \times f$$

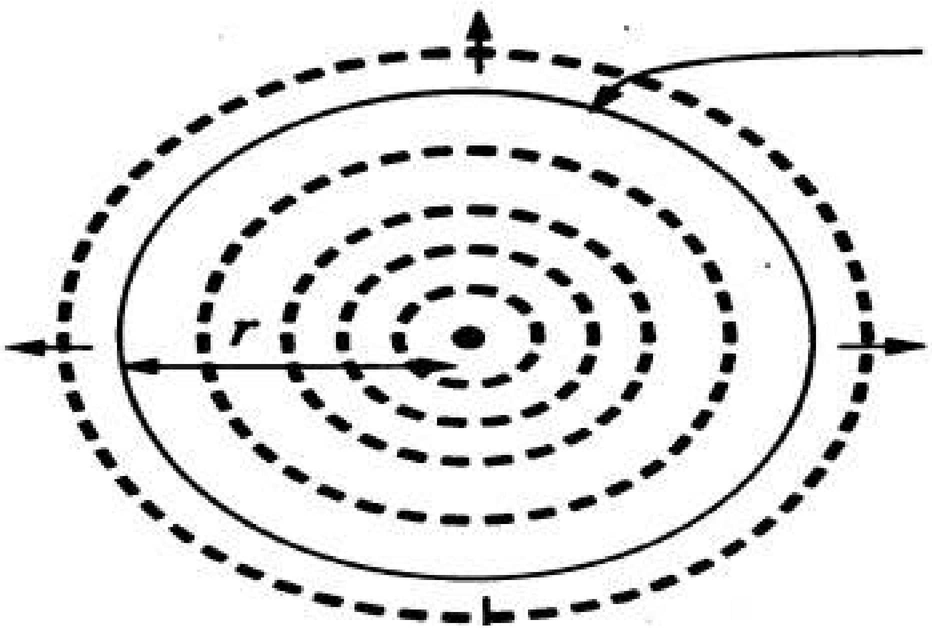

Wavefronts
Ray
(a)
y
λ
E
B
E
dx
h
c
(b)
x
P
B
E
B
z
Electric
component
Magnetic
component

Intensity of EM Radiation

Electromagnetic waves transport *energy*.

The amount of power (i.e. energy per second) transported by an electromagnetic wave per unit surface area is called "intensity" I.

Consider a source which emits radiation power P equally in all directions (i.e. "isotropic"):



Power is distributed uniformly over a sphere of radius r

$$I = \frac{P}{4\pi r^2}$$

POYNTING VECTOR

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

- This is a measure of power per area. Units are watts per meter^2.
- Direction is the direction in which the wave is moving.

POYNTING VECTOR

•However, since **E** and **B** are perpendicular,

$$S = \frac{1}{\mu_0} EB$$

and since $$\frac{E}{B} = c$$

$$S = \frac{1}{c\mu_0} E^2 = \frac{c}{\mu_0} B^2$$

MAXWELL’S EQUATIONS

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \qquad \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0\left(i + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

AMPERE'S LAW

Original:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i$$

As modified by Maxwell:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \left(i + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Reasons for the Extra Term

- SYMMETRY
- CONTINUITY

SYMMETRY

- A time varying magnetic field produces an electric field.
- A time varying electric field produces a magnetic field.

SYMMETRY

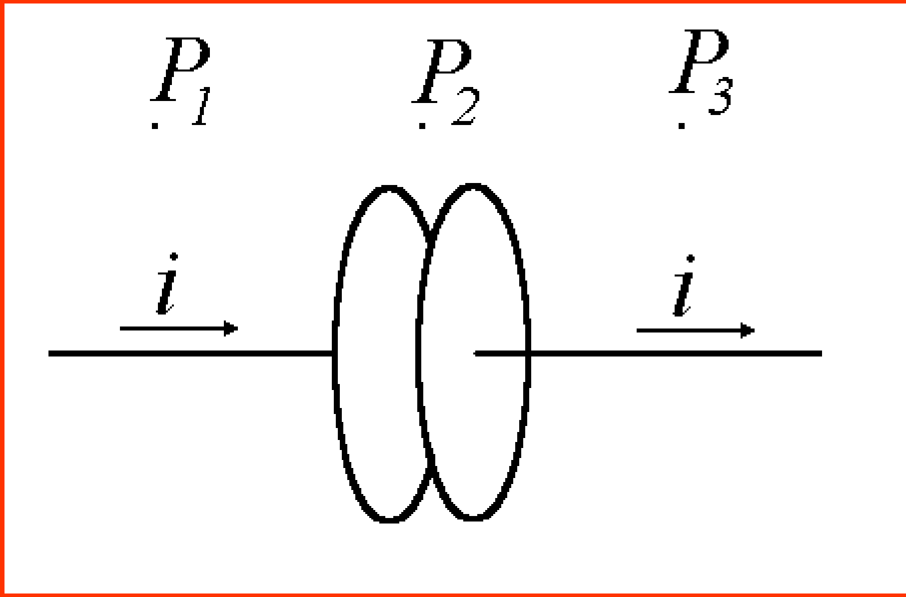
Maxwell's Equations in Free Space

$$q = 0 \qquad\qquad i = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = 0 \qquad\qquad \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad\qquad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

CONTINUITY



CONTINUITY

Without Maxwell's modification:

At P_1: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i$

At P_2: $\oint \mathbf{B} \cdot d\mathbf{s} = 0$

At P_3: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i$

CONTINUITY

With Maxwell's modification:

At P_1: $\oint \mathbf{B} \cdot d\mathbf{s} = \text{«}_0 i$

At P_2: $\oint \mathbf{B} \cdot d\mathbf{s} = \text{«}_0 \text{£}_0 \dfrac{d\Phi_E}{dt} = \text{«}_0 i_d$

At P_3: $\oint \mathbf{B} \cdot d\mathbf{s} = \text{«}_0 i$

$\text{£}_0 \dfrac{d\Phi_E}{dt} = i_d$ "Displacement current"

CONTINUITY

$$i = i_d$$

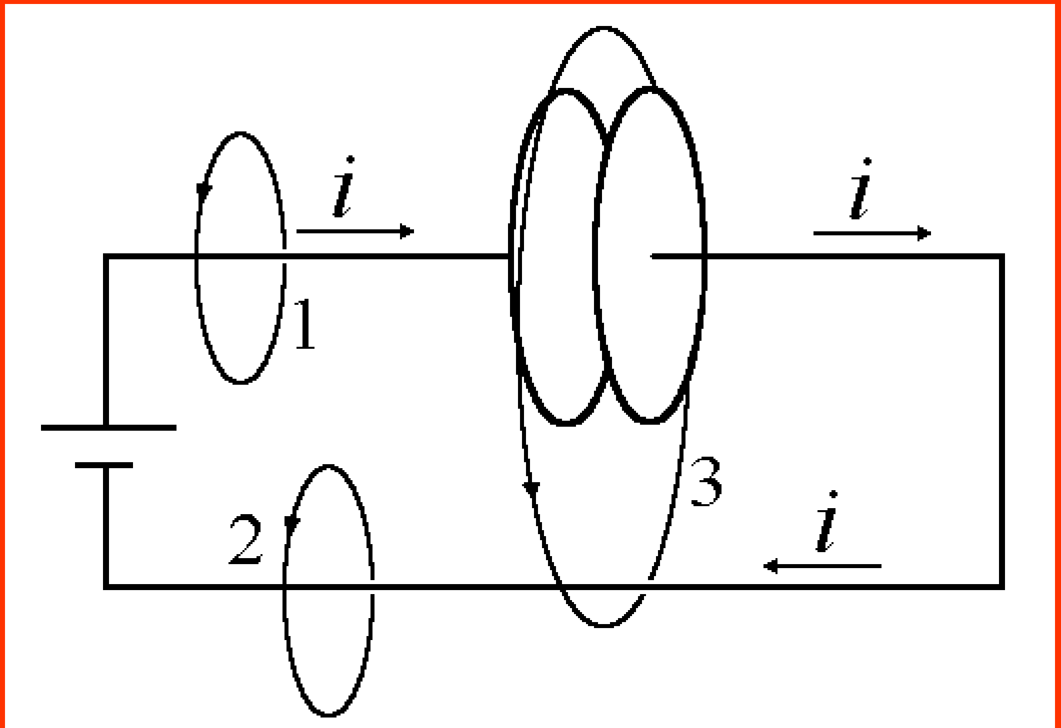
$$q = CV$$

$$C = \pounds_0 \frac{A}{d} \qquad V = Ed$$

$$q = \pounds_0 \frac{A}{d} Ed = \pounds_0 AE = \pounds_0 \Phi_E$$

$$i = \frac{dq}{dt} = \pounds_0 \frac{d\Phi_E}{dt} = i_d$$

AMPERE’S LAW



AMPERE'S LAW

For path 1: $\oint \mathbf{B} \cdot d\mathbf{s} = «_0 i$

For path 2: $\oint \mathbf{B} \cdot d\mathbf{s} = «_0(-i) = -«_0 i$

For path 3: $\oint \mathbf{B} \cdot d\mathbf{s} = «_0(i_d - i) = 0$

since $i_d = i$

ELECTROMAGNETIC WAVES

Maxwell's Equations in Free Space

$$q = 0 \qquad\qquad i = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = 0 \qquad\qquad \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad\qquad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

ELECTROMAGNETIC WAVES

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \qquad \frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

$$\frac{E_m}{B_m} = \frac{E}{B} = c$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \times 10^8 \text{ m/s}$$

MAXWELL'S EQUATIONS

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \qquad \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \left(i + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

$$= \mu_0 (i + i_d)$$

TRANSMISSION LINES AND WAVEGUIDES

References

TEXT BOOK

1. John D Ryder, “Networks lines and fields”, Prentice Hall of India, New Delhi, 2005

REFERENCES

1.William H Hayt and Jr John A Buck, “Engineering Electromagnetics” Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2008

2.David K Cheng, “Field and Wave Electromagnetics”, Pearson Education Inc, Delhi, 2004

3.John D Kraus and Daniel A Fleisch, “Electromagnetics with Applications”, Mc Graw Hill Book Co,2005

4.GSN Raju, “Electromagnetic Field Theory and Transmission Lines”, Pearson Education, 2005

5.Bhag Singh Guru and HR Hiziroglu, “Electromagnetic Field Theory Fundamentals”, Vikas Publishing House, New Delhi, 2001.

6. N. Narayana Rao, “ Elements of Engineering Electromagnetics” 6

Transmission Line

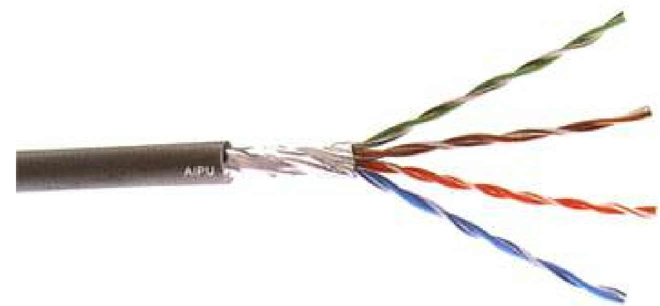
Properties

- Has two conductors running parallel
- Can propagate a signal at any frequency (in theory)
- Becomes lossy at high frequency
- Can handle low or moderate amounts of power
- Does not have signal distortion, unless there is loss
- May or may not be immune to interference
- Does not have E_z or H_z components of the fields (TEM_z)

Coaxial cable (coax)

Twin lead
(shown connected to a 4:1 impedance-transforming balun)

Transmission Line (cont.)

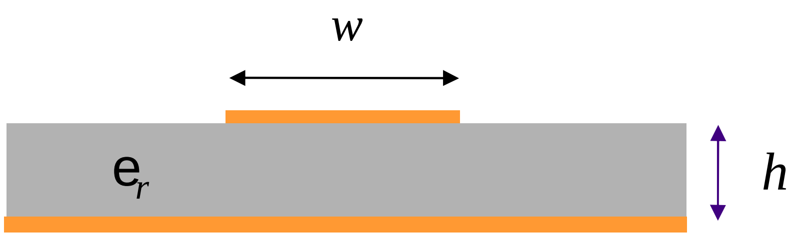


CAT 5 cable
(twisted pair)

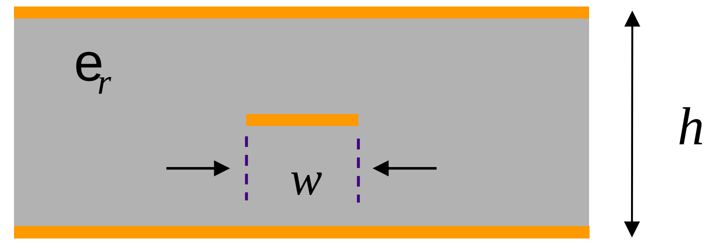
The two wires of the transmission line are twisted to reduce interference and radiation from discontinuities.

Transmission Line (cont.)

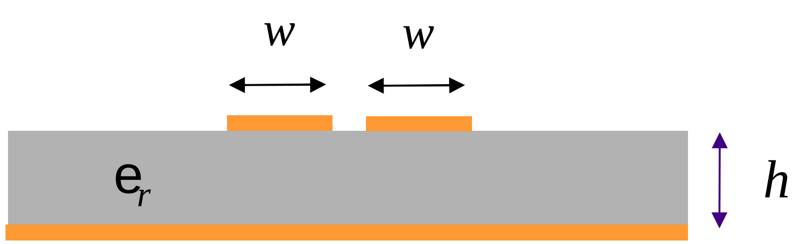
Transmission lines commonly met on printed-circuit boards



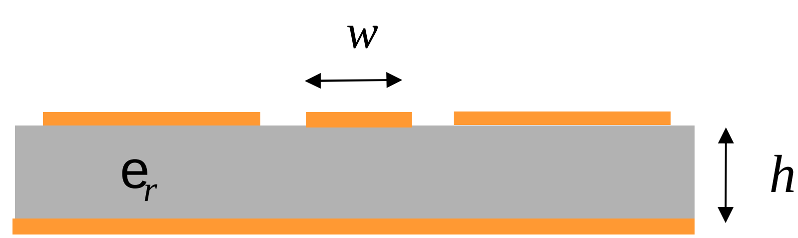
Microstrip



Stripline



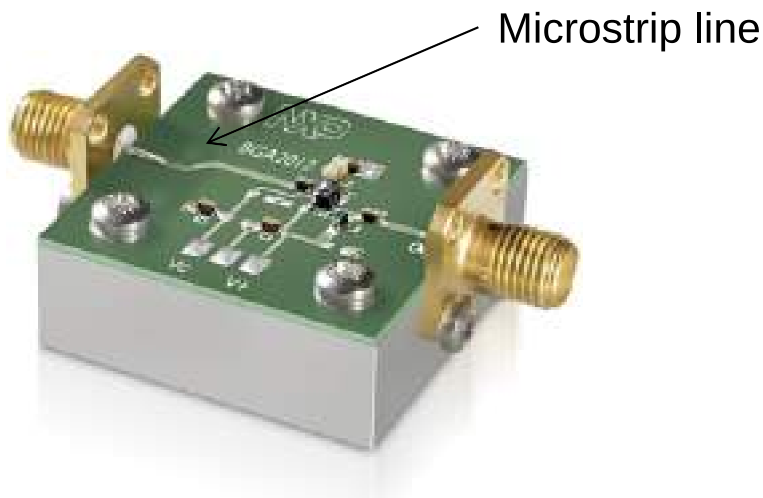
Coplanar strips



Coplanar waveguide (CPW)

Transmission Line (cont.)

Transmission lines are commonly met on printed-circuit boards.

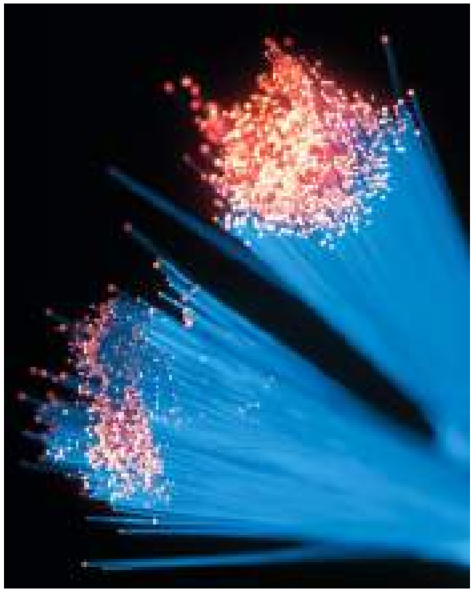


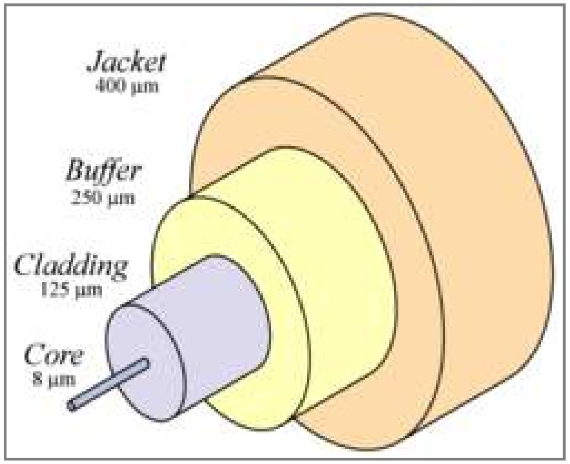
A microwave integrated circuit

Fiber-Optic Guide

Properties

- Uses a dielectric rod
- Can propagate a signal at any frequency (in theory)
- Can be made very low loss
- Has minimal signal distortion
- Very immune to interference
- Not suitable for high power
- Has both E_z and H_z components of the fields





Waveguides

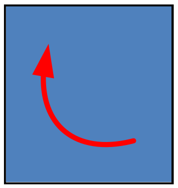
Properties

- Has a single hollow metal pipe
- Can propagate a signal only at high frequency: $> {}_c$
- The width must be at least one-half of a wavelength
- Has signal distortion, even in the lossless case
- Immune to interference
- Can handle large amounts of power
- Has low loss (compared with a transmission line)
- Has either E_z or H_z component of the fields (TM_z or TE_z)

http://en.wikipedia.org/wiki/Waveguide_(electromagnetism)

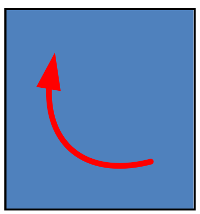
Transmission-Line Theory

- Lumped circuits: resistors, capacitors, inductors



neglect time delays (phase)

- Distributed circuit elements: transmission lines

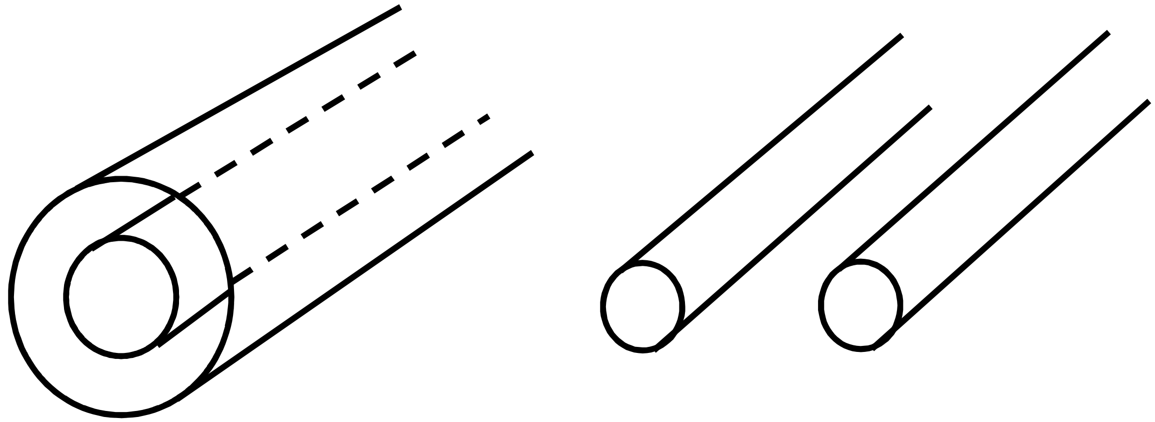


account for propagation and
time delays (phase change)

We need transmission-line theory whenever the length of a line is significant compared with a wavelength.

Transmission Line

2 conductors



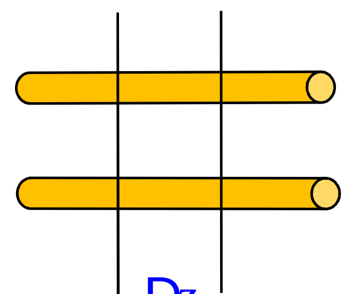
4 per-unit-length parameters:

C = capacitance/length [F/m]

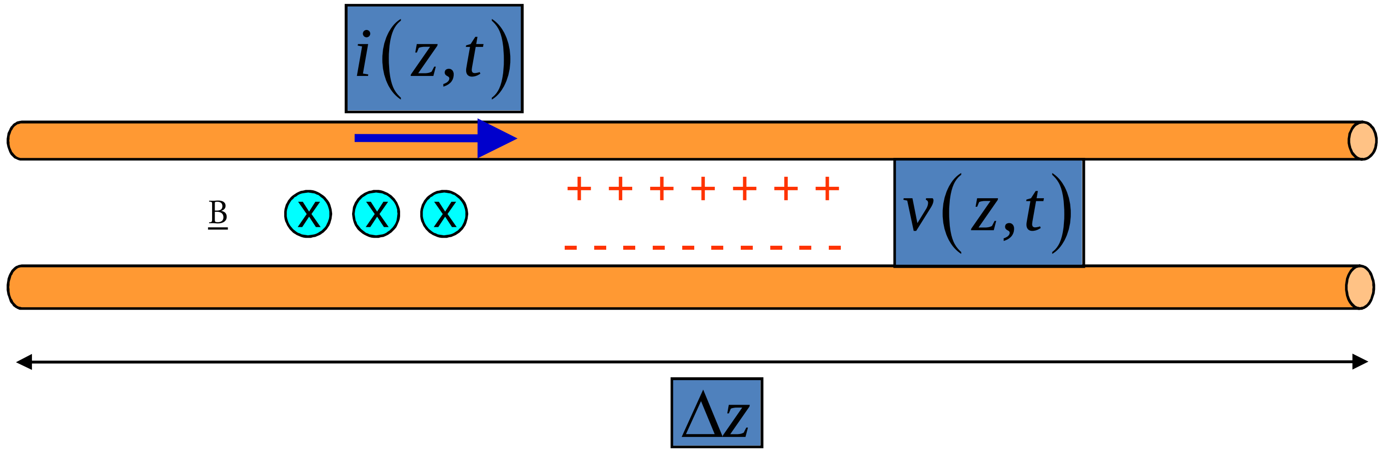
L = inductance/length [H/m]

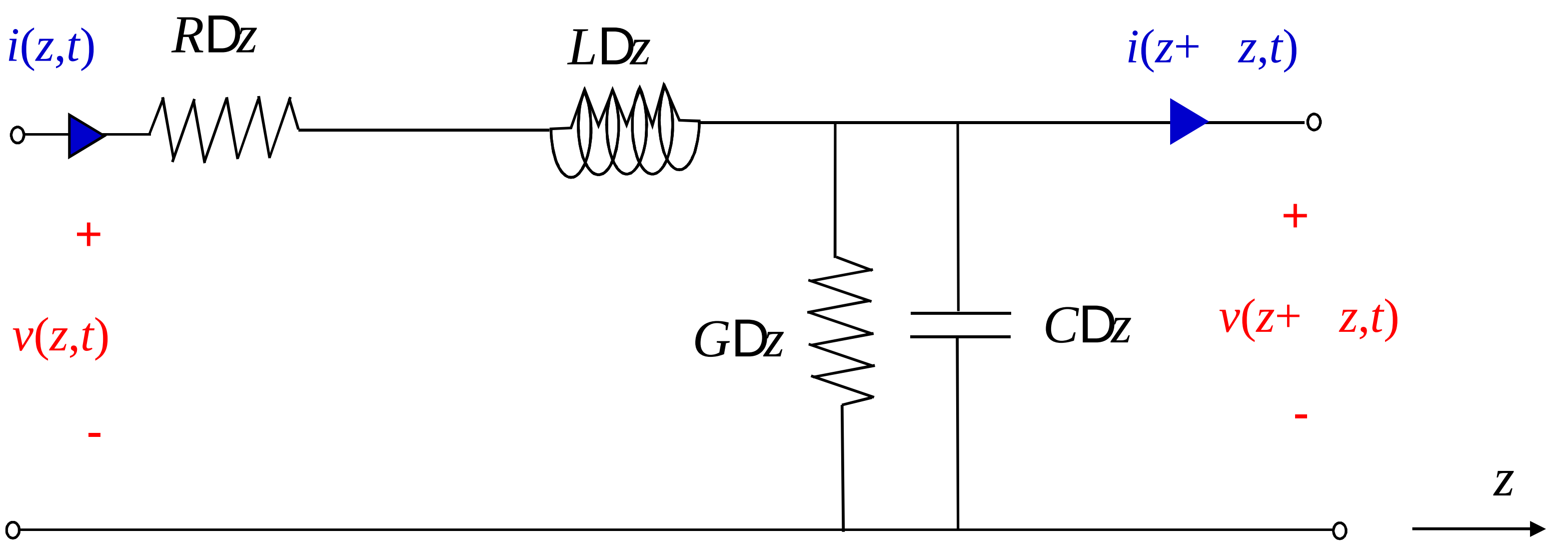
R = resistance/length [Ω/m]

G = conductance/length [℧/m or S/m]



Transmission Line (cont.)





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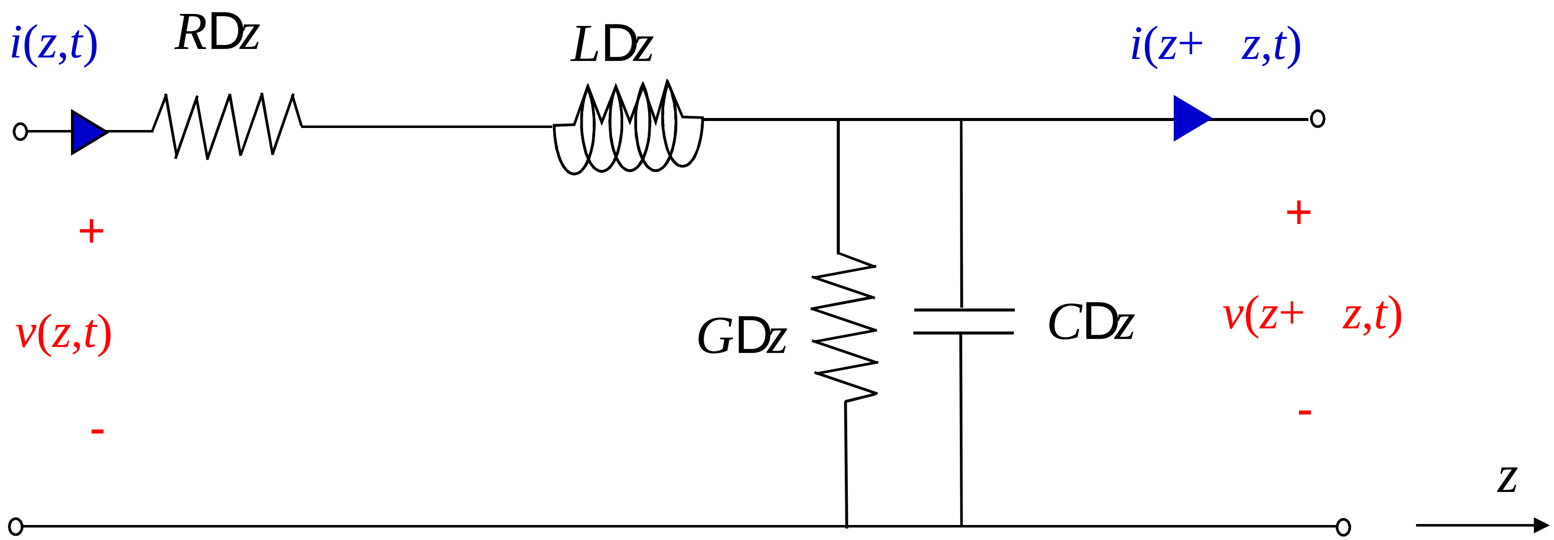
Transmission Line (cont.)



$$v(z,t) = v(z+\Delta z,t) + i(z,t)R\Delta z + L\Delta z\frac{\partial i(z,t)}{\partial t}$$

$$i(z,t) = i(z+\Delta z,t) + v(z+\Delta z,t)G\Delta z + C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t}$$

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TEM Transmission Line (cont.)

Hence

$$\frac{v(z+\Delta z,t)-v(z,t)}{\Delta z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{i(z+\Delta z,t)-i(z,t)}{\Delta z} = -Gv(z+\Delta z,t) - C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

Now let Δz 0:

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z} = -Gv - C\frac{\partial v}{\partial t}$$

"Telegrapher's Equations"

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TEM Transmission Line (cont.)

To combine these, take the derivative of the first one with respect to z:

$$\frac{\partial^2 v}{\partial z^2} = -R\frac{\partial i}{\partial z} - L\frac{\partial}{\partial z}\left(\frac{\partial i}{\partial t}\right)$$

$$= -R\frac{\partial i}{\partial z} - L\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial z}\right)$$

$$= -R\left[-Gv - C\frac{\partial v}{\partial t}\right]$$

$$- L\left[-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right]$$

Switch the order of the derivatives.

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TEM Transmission Line (cont.)

$$\frac{\partial^2 v}{\partial z^2} = -R\left[-Gv - C\frac{\partial v}{\partial t}\right] - L\left[-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right]$$

Hence, we have:

$$\frac{\partial^2 v}{\partial z^2} - (RG)v - (RC + LG)\frac{\partial v}{\partial t} - LC\left(\frac{\partial^2 v}{\partial t^2}\right) = 0$$

The same equation also holds for i.

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TEM Transmission Line (cont.)

Let $¥^{2} = ZY$

Then $\dfrac{d^{2}V}{dz^{2}} = (¥^{2})V$

Solution: $V(z) = Ae^{-¥z} + Be^{+¥z}$

¥ is called the "propagation constant."

Convention: $¥ = \left[(R + j\mu L)(G + j\mu C)\right]^{1/2}$

$=$ principal square root

$\sqrt{z} = \sqrt{|z|}\, e^{j¯/2}$

$-® < ¯ < ®$

$¥ = \sim + j$

$\sim \geq 0, \quad \geq 0$

$\sim$ = attenuation contant

= phase constant

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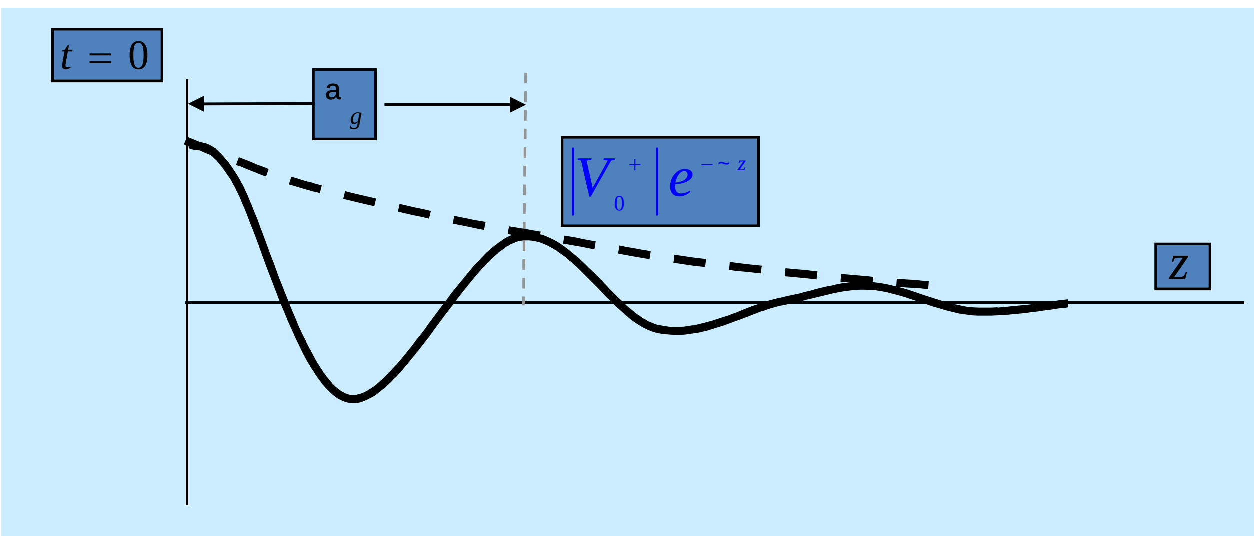
TEM Transmission Line (cont.)

Forward travelling wave (a wave traveling in the positive z direction):

$$V^+(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

$$\begin{aligned} v^+(z,t) &= \operatorname{Re}\left\{\left(V_0^+ e^{-\alpha z} e^{-j\beta z}\right)e^{j\omega t}\right\} \\ &= \operatorname{Re}\left\{\left(\left|V_0^+\right| e^{j\phi} e^{-\alpha z} e^{-j\beta z}\right)e^{j\omega t}\right\} \\ &= \left|V_0^+\right| e^{-\alpha z}\cos\left(\omega t - \beta z + \phi\right) \end{aligned}$$



The wave "repeats" when:

$$\beta \lambda_g = 2\pi$$

Hence:

$$\beta = \frac{2\pi}{\lambda_g}$$

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Phase Velocity (cont.)

Set

$$\mu t - \quad z = \text{constant}$$

$$\mu - \quad \frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \frac{\mu}{\quad}$$

Hence

$$v_p = \frac{\mu}{\quad}$$

In expanded form:

$$v_p = \frac{\mu}{\text{Im}\left\{\left[(R + j\mu L)(G + j\mu C)\right]^{1/2}\right\}}$$

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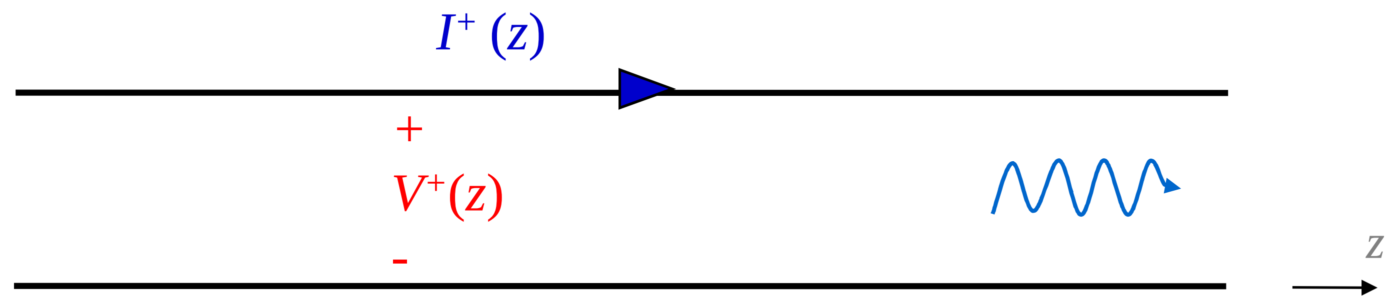
Characteristic Impedance Z_0



A wave is traveling in the positive *z* direction.

$$Z_0 \equiv \frac{V^+(z)}{I^+(z)}$$

$$V^+(z) = V_0^+ e^{-\gamma z}$$

$$I^+(z) = I_0^+ e^{-\gamma z}$$

so

$$Z_0 = \frac{V_0^+}{I_0^+}$$

(Z_0 is a number, not a function of z.)

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Characteristic Impedance Z_0 (cont.)

Use Telegrapher's Equation:

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

so

$$\frac{dV}{dz} = -RI - j\mu LI$$
$$= -ZI$$

Hence

$$-¥V_0^+ e^{-¥z} = -ZI_0^+ e^{-¥z}$$

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Characteristic Impedance Z_0 (cont.)

From this we have:

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{Z}{¥} = \left(\frac{Z}{Y}\right)^{1/2}$$

Using

$$Z = R + j\mu L$$

$$Y = G + j\mu C$$

We have

$$Z_0 = \left(\frac{R + j\mu L}{G + j\mu C}\right)^{1/2}$$

Note: The principal branch of the square root is chosen, so that Re $(Z_0) > 0$.

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General Case (Waves in Both Directions)

$$V(z) = V_0^+ e^{+¥z} + V_0^- e^{+¥z}$$

$$= \left|V_0^+\right| e^{j¤^+} e^{+~z} e^{-j\ z} + \left|V_0^-\right| e^{j¤^-} e^{+~z} e^{+j\ z}$$

Note:

wave in $+z$ direction

wave in $-z$ direction

$$v(z,t) = \mathrm{Re}\left\{V(z) e^{jµt}\right\}$$

$$= \left|V_0^+\right| e^{-~z} \cos\left(µt - \ z + ¤^+\right)$$

$$+ \left|V_0^-\right| e^{+~z} \cos\left(µt + \ z + ¤^-\right)$$

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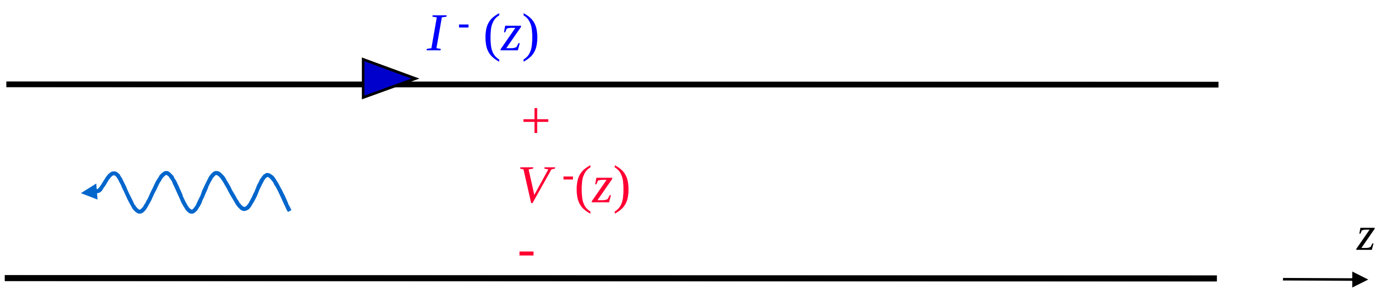
Backward-Traveling Wave



A wave is traveling in the negative z direction.

$$\frac{V^-(z)}{-I^-(z)} = Z_0$$

so

$$\frac{V^-(z)}{I^-(z)} = -Z_0$$

Note: The reference directions for voltage and current are the same as for the forward wave.

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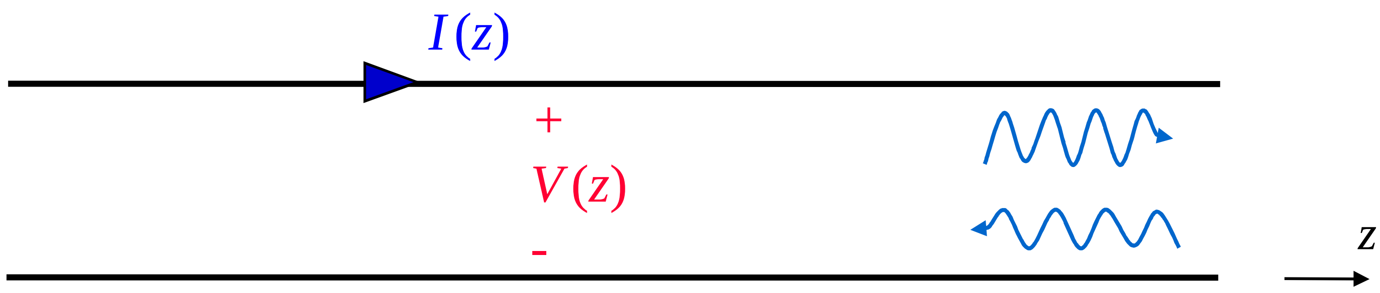
General Case



Most general case:

A general superposition of forward and backward traveling waves:

$$V(z) = V_0^{+} e^{-\gamma z} + V_0^{-} e^{+\gamma z}$$

$$I(z) = \frac{1}{Z_0}\left[V_0^{+} e^{-\gamma z} - V_0^{-} e^{+\gamma z}\right]$$

Note: The reference directions for voltage and current are the same for forward and backward waves.

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Lossless Case

$$R = 0, \; G = 0$$

$$¥ = \sim + j \quad = \left[(R + j\mu L)(G + j\mu C) \right]^{1/2}$$
$$= j\mu \sqrt{LC}$$

so

$$\sim = 0$$
$$= \mu \sqrt{LC}$$

$$v_p = \frac{\mu}{\quad}$$

$$Z_0 = \left(\frac{R + j\mu L}{G + j\mu C} \right)^{1/2} \Rightarrow Z_0 = \sqrt{\frac{L}{C}}$$

(real and indep. of freq.)

$$v_p = \frac{1}{\sqrt{LC}}$$

(indep. of freq.)

Lossless Case (cont.)

$$v_p = \frac{1}{\sqrt{LC}}$$

In the medium between the two conductors is homogeneous (uniform) and is characterized by (,), then we have that

$$LC = «£$$

The speed of light in a dielectric medium is $$c_d = \frac{1}{\sqrt{«£}}$$

Hence, we have that $$v_p = c_d$$

The phase velocity does not depend on the frequency, and it is always the speed of light (in the material).

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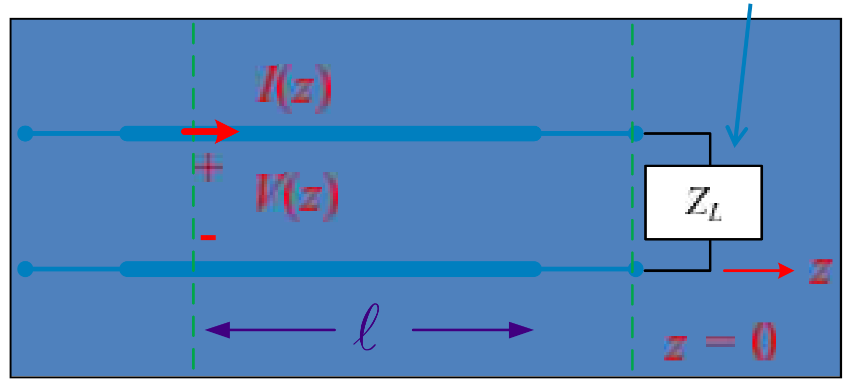
Terminated Transmission Line (cont.)

$$V(z) = V_0^+ e^{-\yen z} + V_0^- e^{+\yen z}$$

Terminating impedance (load)



What if we know

V^+ and V^- $@\ z = -\ell$

Can we use $z = -\ell$ as a reference plane?

$$V_0^+ = V^+(0) = V^+(-\ell)e^{-\yen\ell}$$

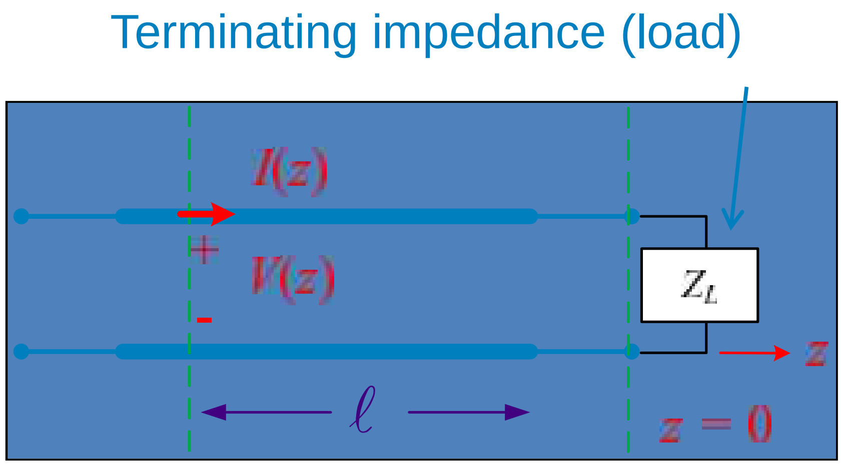
$$V^-(-\ell) = V^-(0)e^{-\yen\ell}$$

$$\Rightarrow V_0^- = V^-(0) = V^-(-\ell)e^{\yen\ell}$$

Hence

$$V(z) = V^+(-\ell)e^{-\yen(z+\ell)} + V^-(-\ell)e^{\yen(z+\ell)}$$

Terminated Transmission Line (cont.)



Compare:

$$V(z) = V^{+}(0)e^{-¥z} + V^{-}(0)e^{+¥z}$$

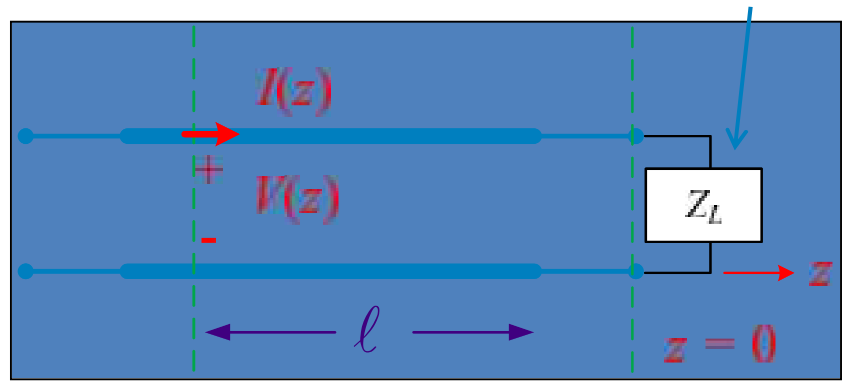
$$V(z) = V^{+}(-\ell)e^{-¥(z-(-\ell))} + V^{-}(-\ell)e^{¥(z-(-\ell))}$$

Note: This is simply a change of reference plane, from $z = 0$ to $z = -\ell$.

Terminated Transmission Line (cont.)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

Terminating impedance (load)



What is $V(-\ell)$?

$$V(-\ell) = V_0^+ e^{\gamma \ell} + V_0^- e^{-\gamma \ell}$$

propagating forwards

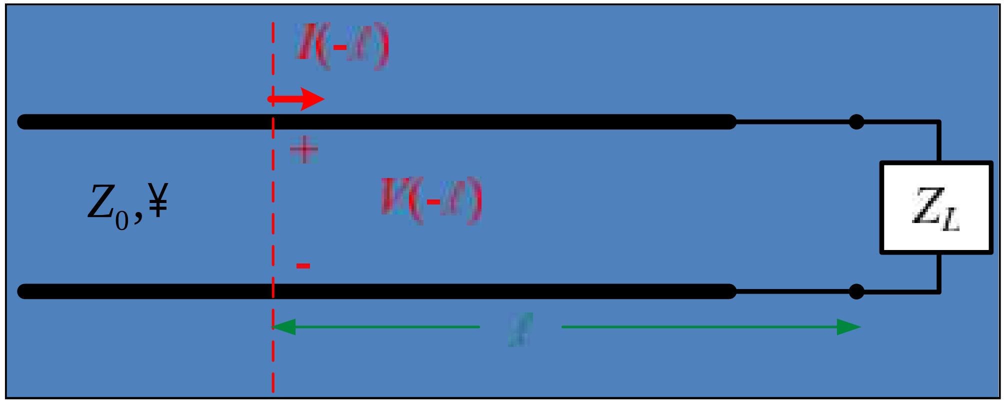
propagating backwards

The current at $z = -\ell$ is then

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma \ell} - \frac{V_0^-}{Z_0} e^{-\gamma \ell}$$

ℓ distance away from load

Terminated Transmission Line (cont.)



Total volt. at distance ℓ from the load

$$V(-\ell) = V_0^+ e^{\gamma \ell} + V_0^- e^{-\gamma \ell} = V_0^+ e^{\gamma \ell}\left(1 + \frac{V_0^-}{V_0^+} e^{-2\gamma \ell}\right)$$

Ampl. of volt. wave prop. towards load, at the load position ($z = 0$).

Ampl. of volt. wave prop. away from load, at the load position ($z = 0$).

L Load reflection coefficient

Reflection coefficient at $z = -\ell$

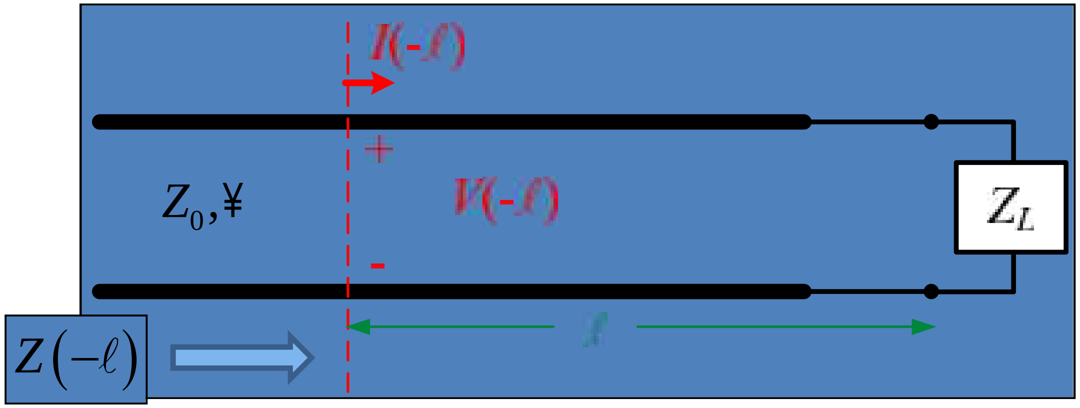
ℓ ℓ

$$= V_0^+ e^{\gamma \ell}\left(1 + \Gamma_L e^{-2\gamma \ell}\right)$$

Similarly,

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma \ell}\left(1 - \Gamma_L e^{-2\gamma \ell}\right)$$

Terminated Transmission Line (cont.)



$$V(-\ell) = V_0^+ e^{¥\ell}\left(1 + \Gamma_L e^{-2¥\ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{¥\ell}\left(1 - \Gamma_L e^{-2¥\ell}\right)$$

$$Z(-\ell) = \frac{V(-\ell)}{I(-\ell)} = Z_0\left(\frac{1 + \Gamma_L e^{-2¥\ell}}{1 - \Gamma_L e^{-2¥\ell}}\right)$$

Input impedance seen "looking" towards load at $z = -\ell$.

Terminated Transmission Line (cont.)

Simplifying, we have

$$Z(-\ell) = Z_0 \left(\frac{1 + \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2¥\ell}}{1 - \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2¥\ell}} \right) = Z_0 \left(\frac{(Z_L + Z_0) + (Z_L - Z_0) e^{-2¥\ell}}{(Z_L + Z_0) - (Z_L - Z_0) e^{-2¥\ell}} \right)$$

$$= Z_0 \left(\frac{(Z_L + Z_0) e^{+¥\ell} + (Z_L - Z_0) e^{-¥\ell}}{(Z_L + Z_0) e^{+¥\ell} - (Z_L - Z_0) e^{-¥\ell}} \right)$$

$$= Z_0 \left(\frac{Z_L \cosh(¥\ell) + Z_0 \sinh(¥\ell)}{Z_0 \cosh(¥\ell) + Z_L \sinh(¥\ell)} \right)$$

Hence, we have

$$Z(-\ell) = Z_0 \left(\frac{Z_L + Z_0 \tanh(¥\ell)}{Z_0 + Z_L \tanh(¥\ell)} \right)$$

Terminated Lossless Transmission Line

$$\text{¥} = \sim + j \quad = j$$

$$V(-\ell) = V_0^+ e^{j \ \ell}\left(1 + \Gamma_L e^{-2j \ \ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{j \ \ell}\left(1 - \Gamma_L e^{-2j \ \ell}\right)$$

$$Z(-\ell) = Z_0 \left(\frac{1 + \Gamma_L e^{-2j \ \ell}}{1 - \Gamma_L e^{-2j \ \ell}}\right)$$

Impedance is periodic with period $_g/2$

tan repeats when

$$\ell = \text{®}$$

$$\frac{2\text{®}}{\text{a}_g}\ell = \text{®}$$

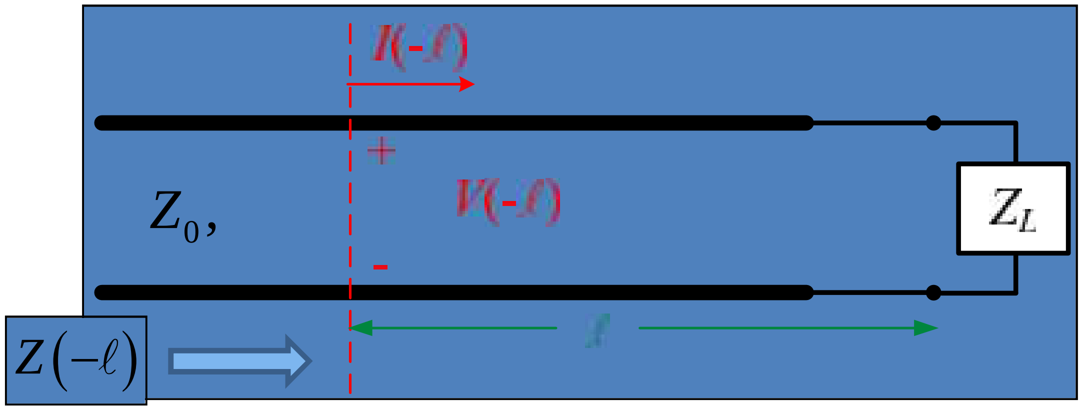
$$\Rightarrow \ell = \text{a}_g / 2$$

$$Z(-\ell) = Z_0 \left(\frac{Z_L + jZ_0 \tan(\ \ell)}{Z_0 + jZ_L \tan(\ \ell)}\right)$$

Note: $\tanh(\text{¥}\ell) = \tanh(j \ \ell) = j \tan(\ \ell)$

Terminated Lossless Transmission Line

For the remainder of our transmission line discussion we will assume that the transmission line is lossless.



$$V(-\ell) = V_0^+ e^{j \ \ell}\left(1+\Gamma_L e^{-2j \ \ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{j \ \ell}\left(1-\Gamma_L e^{-2j \ \ell}\right)$$

$$Z(-\ell) = \frac{V(-\ell)}{I(-\ell)} = Z_0\left(\frac{1+\Gamma_L e^{-2j \ \ell}}{1-\Gamma_L e^{-2j \ \ell}}\right)$$

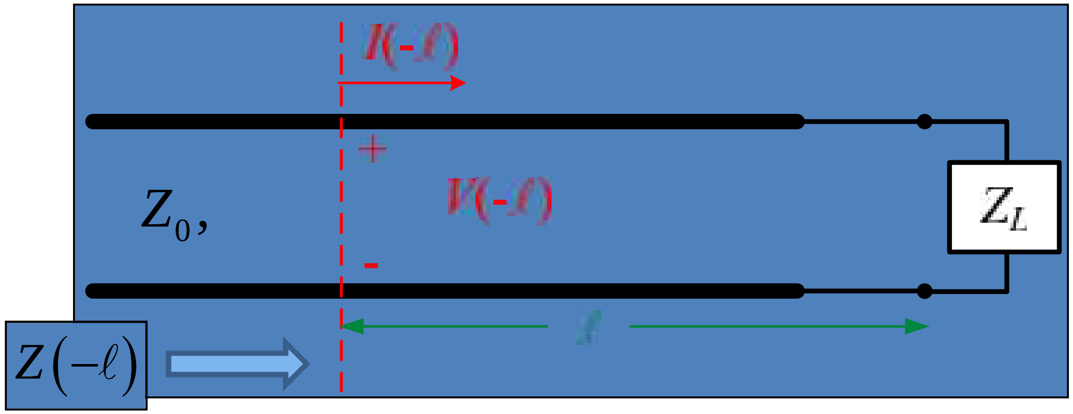
$$= Z_0\left(\frac{Z_L + jZ_0 \tan(\ \ell)}{Z_0 + jZ_L \tan(\ \ell)}\right)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

a$_g$ = 2® /

v_p = µ /

Matched Load



Ⓐ Matched load: ($Z_L = Z_0$)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

No reflection from the load

$$\Rightarrow V(-\ell) = V_0^{+} e^{+j\ \ell}$$

$$I(-\ell) = \frac{V_0^{+}}{Z_0} e^{+j\ \ell}$$

$$\Rightarrow Z(-\ell) = Z_0$$

For any ℓ

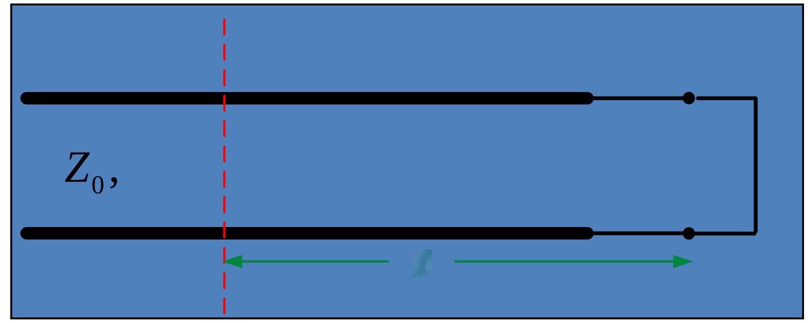
Short-Circuit Load

(B) Short circuit load: ($Z_L = 0$)

$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = -1$$

$$\Rightarrow Z(-\ell) = jZ_0 \tan(\ \ell)$$

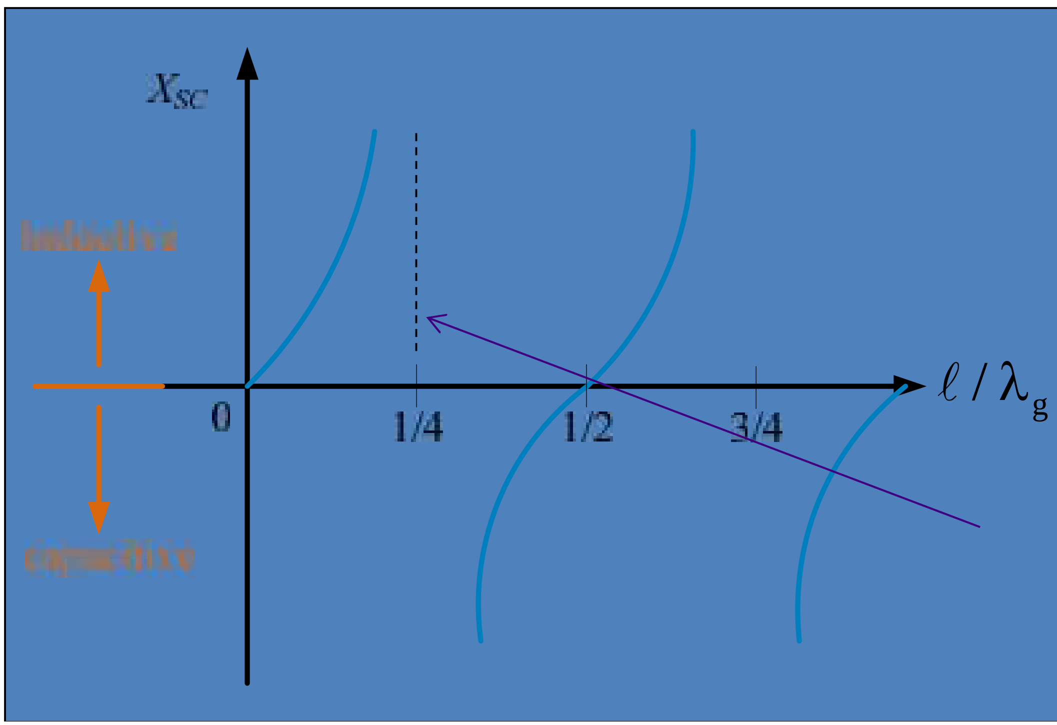
Always imaginary!



Note: $\ell = 2$® $\frac{\ell}{a_g}$

$$\Rightarrow Z(-\ell) = jX_{sc}$$

$$X_{sc} = Z_0 \tan(\ \ell)$$

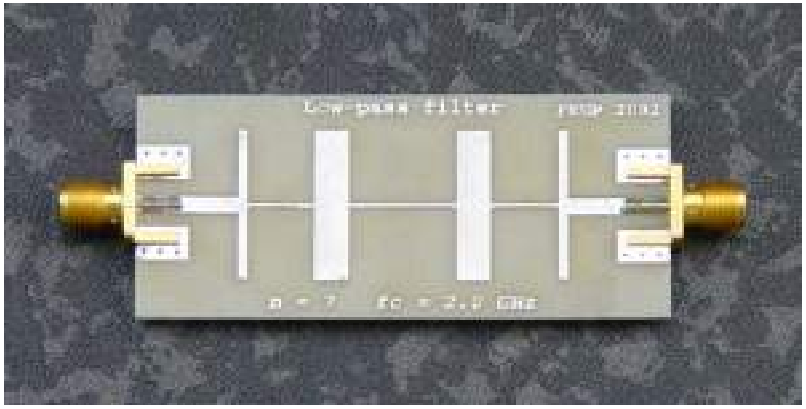


C. can become an O.C.

h a $_g$/4 trans. line

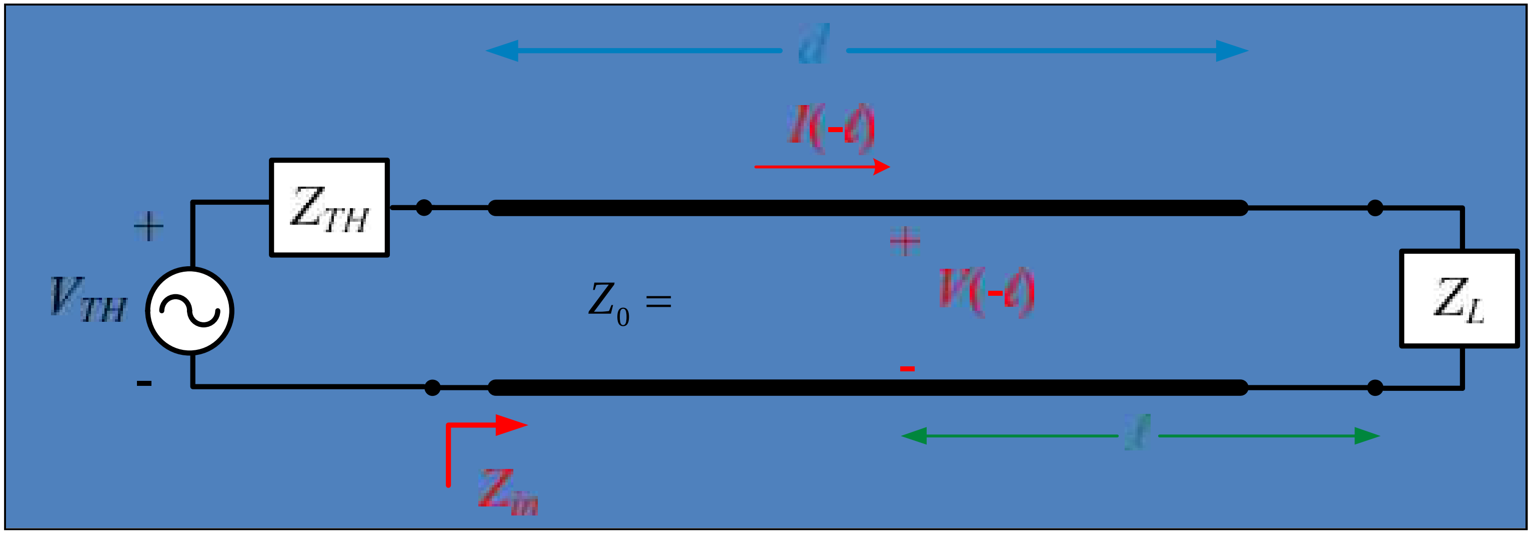
Using Transmission Lines to Synthesize Loads

This is very useful is microwave engineering.



A microwave filter constructed from microstrip.

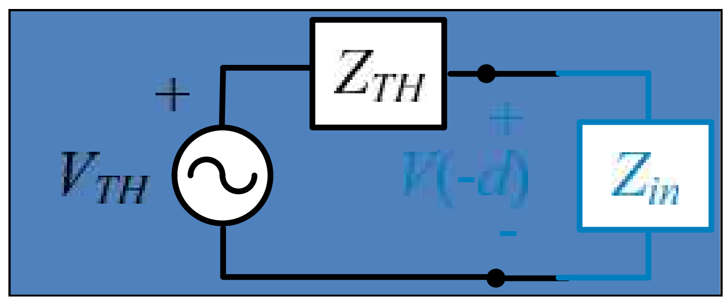
Example



Find the voltage at any point on the line.

$$Z_{in} = Z(-d) = Z_0 \left(\frac{Z_L + jZ_0 \tan(\ d)}{Z_0 + jZ_L \tan(\ d)} \right)$$

$$\Rightarrow \ V(-d) = V_{TH} \left(\frac{Z_{in}}{Z_{in} + Z_{TH}} \right)$$



Example (cont.)

Note: $$V(-\ell) = V_0^+ e^{j\ \ell}\left(1 + \Gamma_L e^{-2j\ \ell}\right)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

At $\ell = d$:

$$V(-d) = V_0^+ e^{j\ d}\left(1 + \Gamma_L e^{-j2\ d}\right) = V_{TH}\left(\frac{Z_{in}}{Z_{in} + Z_{TH}}\right)$$

$$\Rightarrow V_0^+ = V_{TH}\left(\frac{Z_{in}}{Z_{in} + Z_{TH}}\right) e^{-j\ d}\left(\frac{1}{1 + \Gamma_L e^{-j2\ d}}\right)$$

Hence

$$V(-\ell) = V_{TH}\left(\frac{Z_{in}}{Z_m + Z_{TH}}\right) e^{-j\ (d-\ell)}\left(\frac{1 + \Gamma_L e^{-j2\ \ell}}{1 + \Gamma_L e^{-j2\ d}}\right)$$

Example (cont.)

Some algebra:

$$Z_{in} = Z(-d) = Z_0\left(\frac{1+\Gamma_L e^{-j2\;d}}{1-\Gamma_L e^{-j2\;d}}\right)$$

$$\Rightarrow \frac{Z_{in}}{Z_{in}+Z_{TH}} = \frac{Z_0\left(\dfrac{1+\Gamma_L e^{-j2\;d}}{1-\Gamma_L e^{-j2\;d}}\right)}{Z_0\left(\dfrac{1+\Gamma_L e^{-j2\;d}}{1-\Gamma_L e^{-j2\;d}}\right)+Z_{TH}} = \frac{Z_0\left(1+\Gamma_L e^{-j2\;d}\right)}{Z_0\left(1+\Gamma_L e^{-j2\;d}\right)+Z_{TH}\left(1-\Gamma_L e^{-j2\;d}\right)}$$

$$= \frac{Z_0\left(1+\Gamma_L e^{-j2\;d}\right)}{\left(Z_{TH}+Z_0\right)+\Gamma_L e^{-j2\;d}\left(Z_0-Z_{TH}\right)}$$

$$= \left(\frac{Z_0}{Z_{TH}+Z_0}\right)\frac{\left(1+\Gamma_L e^{-j2\;d}\right)}{1+\Gamma_L e^{-j2\;d}\left(\dfrac{Z_0-Z_{TH}}{Z_{TH}+Z_0}\right)}$$

$$= \left(\frac{Z_0}{Z_{TH}+Z_0}\right)\frac{\left(1+\Gamma_L e^{-j2\;d}\right)}{1-\Gamma_L e^{-j2\;d}\left(\dfrac{Z_{TH}-Z_0}{Z_{TH}+Z_0}\right)}$$

Example (cont.)

Hence, we have

$$\frac{Z_{in}}{Z_{in}+Z_{TH}}=\left(\frac{Z_0}{Z_0+Z_{TH}}\right)\left(\frac{1+\Gamma_L e^{-j2\ \ d}}{1-\Gamma_S\Gamma_L e^{-j2\ \ d}}\right)$$

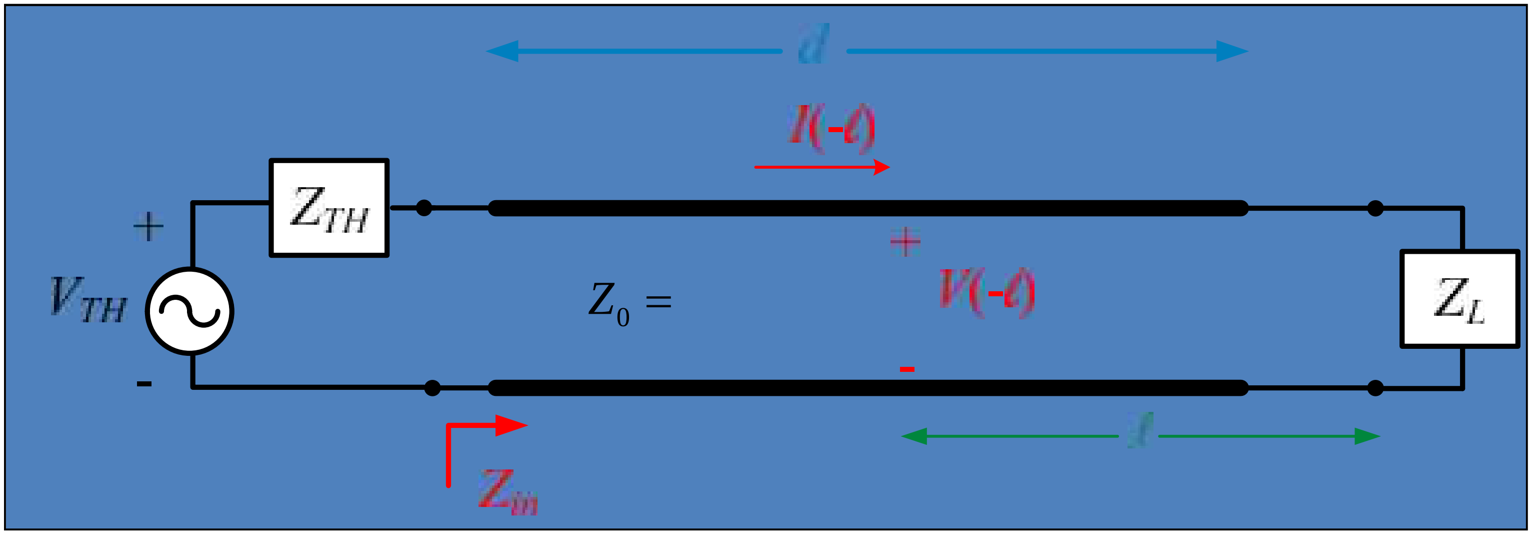
where

$$\Gamma_S=\frac{Z_{TH}-Z_0}{Z_{TH}+Z_0}$$

Therefore, we have the following alternative form for the result:

$$V(-\ell)=V_{TH}\left(\frac{Z_0}{Z_0+Z_{TH}}\right)e^{-j\ \ (d-\ell)}\left(\frac{1+\Gamma_L e^{-j2\ \ \ell}}{1-\Gamma_S\Gamma_L e^{-j2\ \ d}}\right)$$

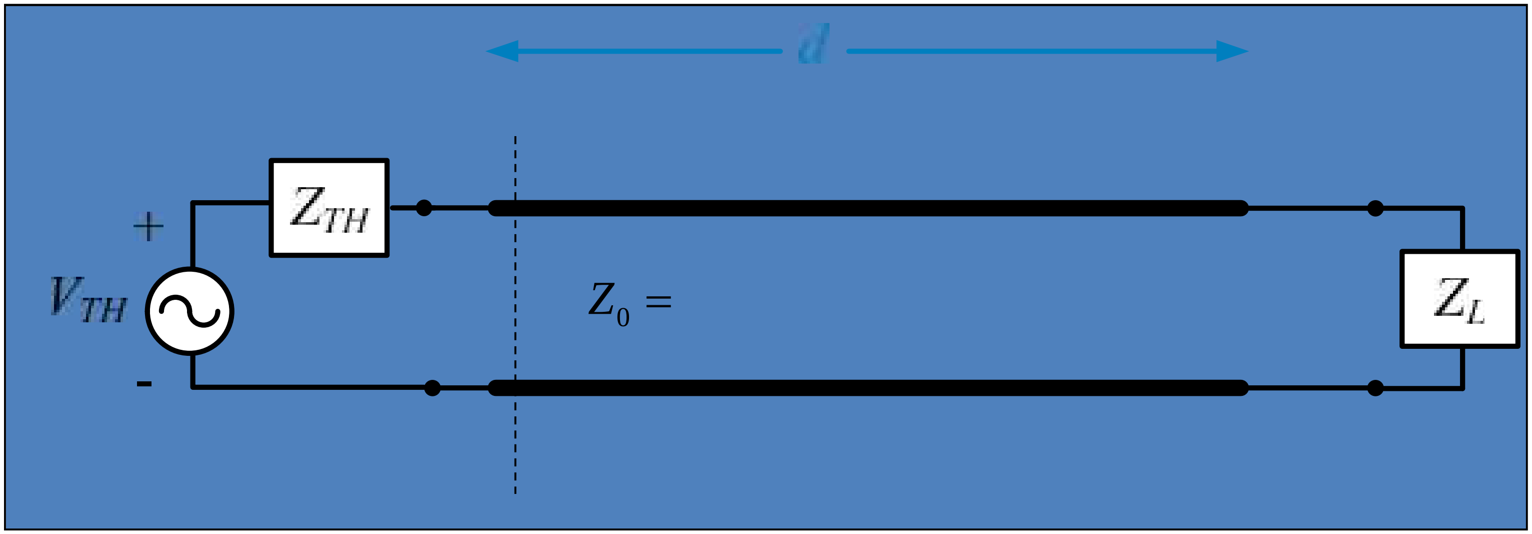
Example (cont.)



$$V(-\ell) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right) e^{-j\beta(d-\ell)} \left(\frac{1+\Gamma_L e^{-j2\beta\ell}}{1-\Gamma_S\Gamma_L e^{-j2\beta d}}\right)$$

Voltage wave that would exist if there were no reflections from the load (a semi-infinite transmission line or a matched load).

Example (cont.)



Wave-bounce method (illustrated for $\ell = d$):

$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\begin{bmatrix} 1 + \Gamma_L e^{-j2\ d} + \left(\Gamma_L e^{-j2\ d}\right)\Gamma_S \\ + \left[\left(\Gamma_L e^{-j2\ d}\right)\Gamma_S\right]\left(\Gamma_L e^{-j2\ d}\right) + \left[\left(\Gamma_L e^{-j2\ d}\right)\Gamma_S\left(\Gamma_L e^{-j2\ d}\right)\right]\Gamma_S \\ + \ldots \end{bmatrix}$$

Example (cont.)

$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\begin{bmatrix} 1 + \Gamma_L e^{-j2\;\,d} + \left(\Gamma_L e^{-j2\;\,d}\right)\Gamma_S \\ + \left[\left(\Gamma_L e^{-j2\;\,d}\right)\Gamma_S\right]\left(\Gamma_L e^{-j2\;\,d}\right) + \left[\left(\Gamma_L e^{-j2\;\,d}\right)\Gamma_S\left(\Gamma_L e^{-j2\;\,d}\right)\right]\Gamma_S \\ + \ldots \end{bmatrix}$$

$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\begin{bmatrix} 1 + \left(\Gamma_L \Gamma_S e^{-j2\;\,d}\right) + \left(\Gamma_L \Gamma_S e^{-j2\;\,d}\right)^2 + \ldots \\ + \Gamma_L e^{-j2\;\,d}\left[1 + \left(\Gamma_L \Gamma_S e^{-j2\;\,d}\right) + \left(\Gamma_L \Gamma_S e^{-j2\;\,d}\right)^2 + \ldots\right] \\ + \ldots \end{bmatrix}$$

Geometric series:

$$\sum_{n=0}^{\infty} z^n = 1 + z + z^2 + \ldots = \frac{1}{1-z}, \quad |z| < 1$$

$$z = \Gamma_L \Gamma_S \, e^{-j2\;\,d}$$

Example (cont.)

Hence

$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\left[\frac{1}{1-\Gamma_L\Gamma_s e^{-j2\ d}} + \Gamma_L e^{-j2\ d}\left(\frac{1}{1-\Gamma_L\Gamma_s e^{-j2\ d}}\right)\right]$$

or

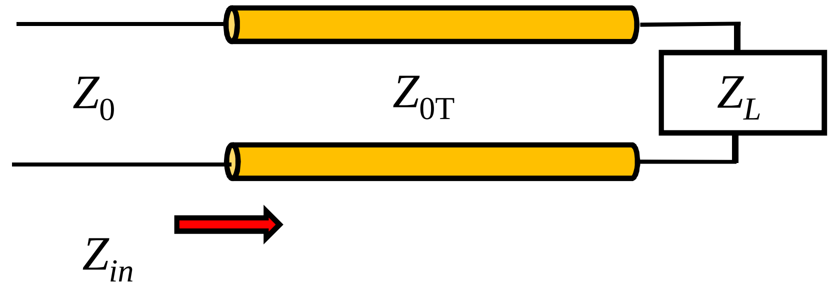
$$V(-d) = V_{TH}\left(\frac{Z_0}{Z_0 + Z_{TH}}\right)\left[\frac{1+\Gamma_L e^{-j2\ d}}{1-\Gamma_L\Gamma_s e^{-j2\ d}}\right]$$

This agrees with the previous result (setting $\ell = d$).

Note: This is a very tedious method – not recommended.

Quarter-Wave Transformer

$$Z_{in} = Z_{0T}\left(\frac{Z_L + jZ_{0T}\tan\ \ell}{Z_{0T} + jZ_L\tan\ \ell}\right)$$



$$\ell = \frac{\lambda_g}{4} = \frac{2\pi}{\lambda_g}\frac{\lambda_g}{4} = \frac{\pi}{2}$$

$$\Rightarrow Z_{in} = Z_{0T}\left(\frac{jZ_{0T}}{jZ_L}\right)$$

so

$$Z_{in} = \frac{Z_{0T}^2}{Z_L}$$

$$\Gamma_{in} = 0 \quad \Rightarrow \quad Z_{in} = Z_0$$

$$\Rightarrow \quad Z_0 = \frac{Z_{0T}^2}{Z_L}$$

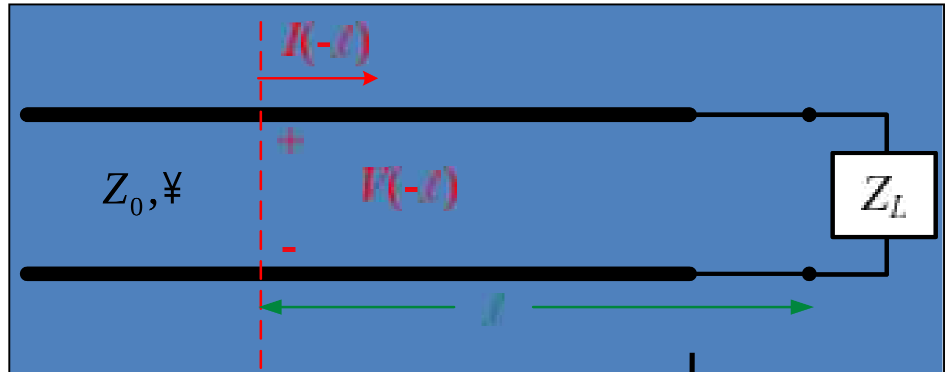
This requires Z_L to be real.

Hence

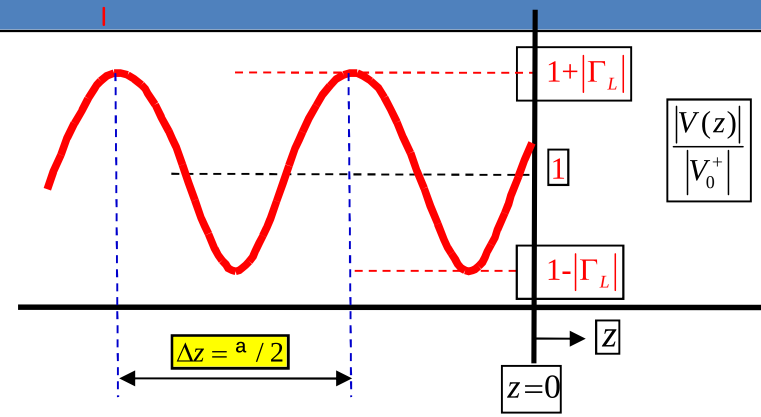
$$Z_{0T} = \left[Z_0 Z_L\right]^{1/2}$$

Voltage Standing Wave Ratio

$$V(-\ell) = V_0^+ e^{j\ \ell}\left(1+\Gamma_L e^{-2j\ \ell}\right)$$
$$= V_0^+ e^{j\ \ell}\left(1+|\Gamma_L| e^{j¤_L} e^{-2j\ \ell}\right)$$



$$|V(-\ell)| = |V_0^+|\,\left|1+|\Gamma_L| e^{j¤_L} e^{-j2\ \ell}\right|$$



$$V_{max} = |V_0^+|\left(1+|\Gamma_L|\right)$$
$$V_{min} = |V_0^+|\left(1-|\Gamma_L|\right)$$

$$\text{Voltage Standing Wave Ratio (VSWR)} = \frac{V_{max}}{V_{min}}$$

$$\text{VSWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

General Transmission Line Formulas

(1) $$\sqrt{\frac{L}{C}} = Z_0^{lossless} = \text{characteristic impedance of line (neglecting loss)}$$

(2) $$LC = \text{«£}' = \text{«}_0\text{£}_0\left(\text{«}_r\text{£}'_r\right)$$

(3) $$\frac{G}{\mu C} = \tan \text{¢}$$

(4) $$R = R_a + R_b$$

$$R_i = R_s\left[\frac{1}{|I|^2}\oint_{C_i} |J_{sz}(l)|^2\, dl\right]$$

$$C_i = \text{contour of conductor},\ i = a,\, b$$

Equations (1) and (2) can be used to find L and C if we know the material properties and the characteristic impedance of the lossless line.

Equation (3) can be used to find G if we know the material loss tangent.

Equation (4) can be used to find R (discussed later).

General Transmission Line Formulas (cont.)

Al four per-unit-length parameters can be found from $Z_0^{lossless},\ R$

$$L = Z_0^{lossless}\,\sqrt{«£'}$$

$$C = \sqrt{«£'}\,/\,Z_0^{lossless}$$

$$G = \left(\mu C\right)\tan ¢$$

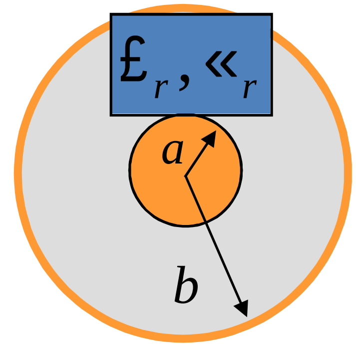
$$R = R$$

Common Transmission Lines

Coax

$$Z_0^{lossless} = ¦_0 \sqrt{\frac{«_r}{£_r}} \frac{1}{2®} \ln\left(\frac{b}{a}\right) \quad [\Omega]$$

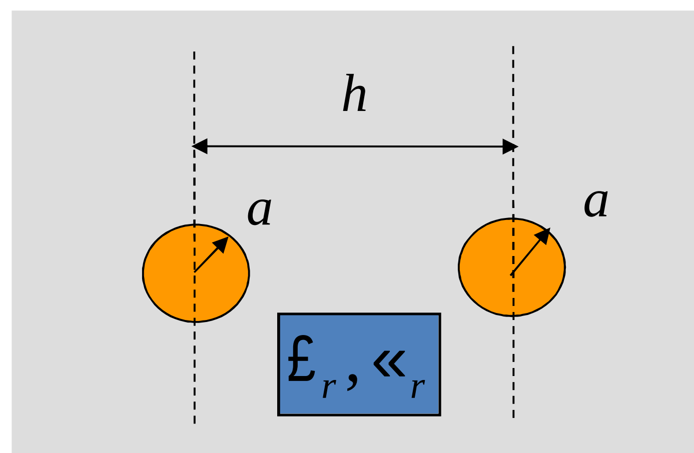
$$R = R_{sa}\left(\frac{1}{2® a}\right) + R_{sb}\left(\frac{1}{2® b}\right)$$



Twin-lead

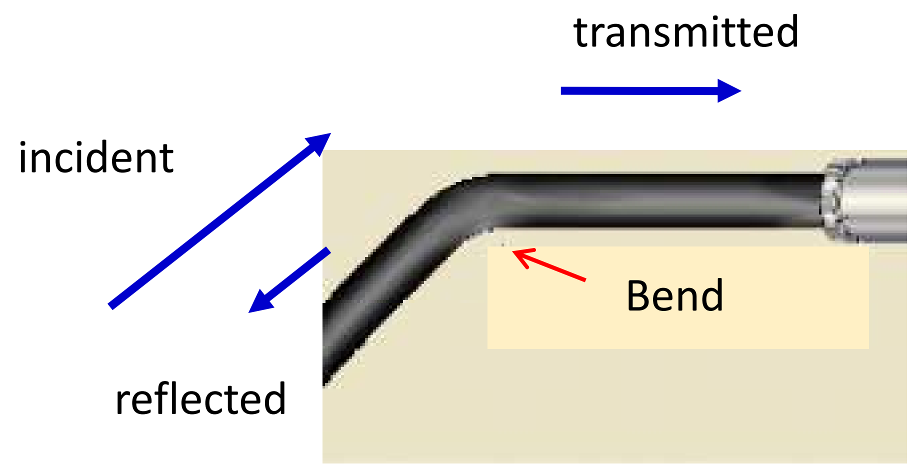
$$Z_0^{lossless} = \frac{¦_0}{®} \sqrt{\frac{«_r}{£_r}} \cosh^{-1}\left(\frac{h}{2a}\right) \quad [\Omega]$$

$$R = R_s \left[\frac{1}{® a} \frac{\left(\frac{h}{2a}\right)}{\sqrt{\left(\frac{h}{2a}\right)^2 - 1}} \right]$$

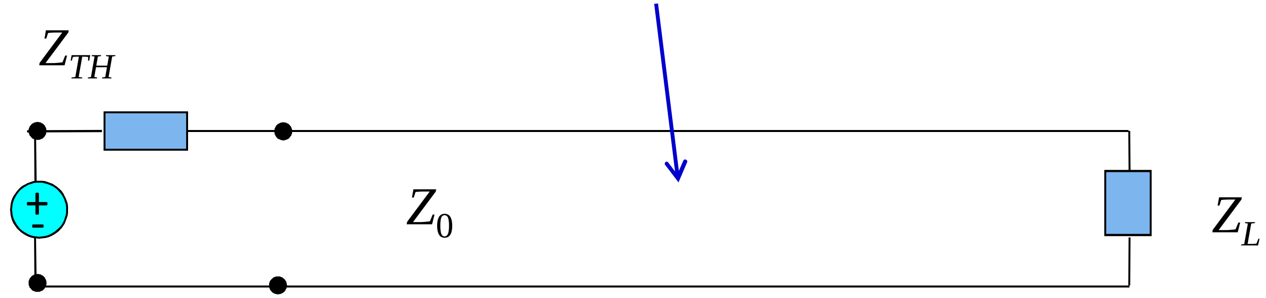


Limitations of Transmission-Line Theory

At high frequency, discontinuity effects can become important.



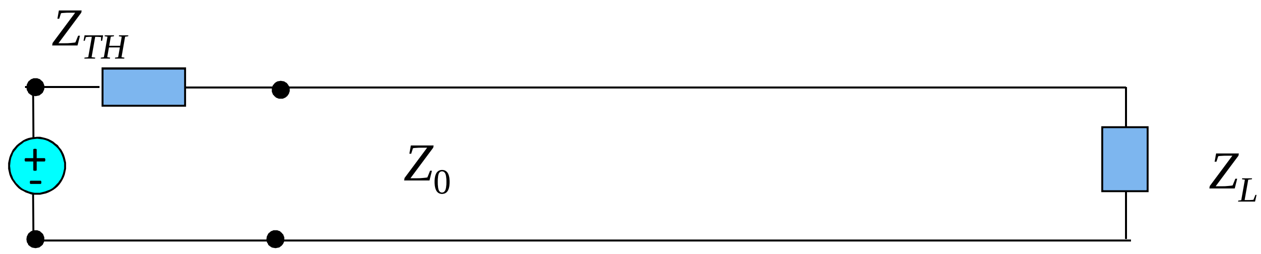
The simple TL model does not account for the bend.



Limitations of Transmission-Line Theory (cont.)

At high frequency, radiation effects can become important.

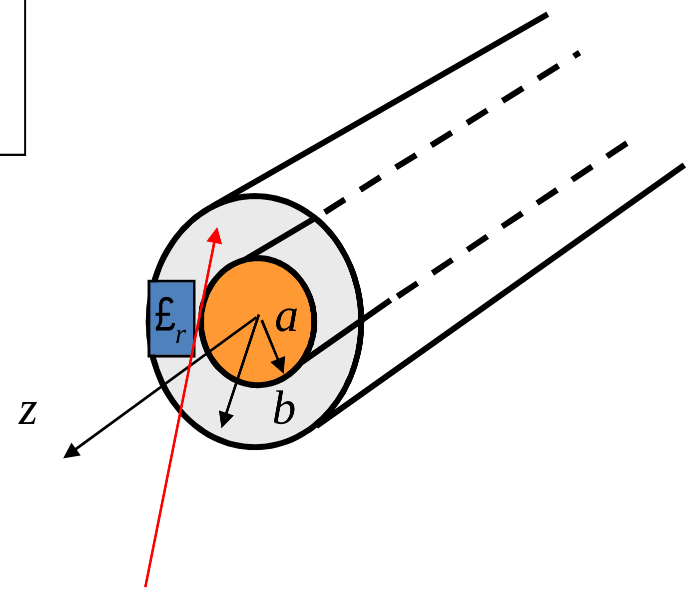
We want energy to travel from the generator to the load, without radiating.



When will radiation occur?

Limitations of Transmission-Line Theory (cont.)

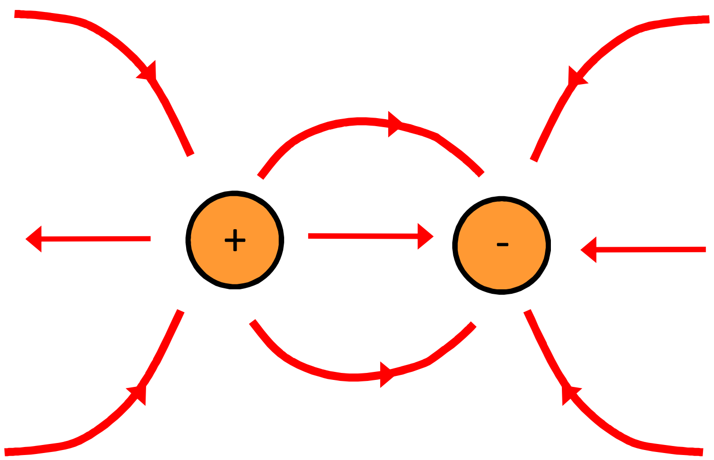
The coaxial cable is a perfectly shielded system – there is never any radiation at any frequency, or under any circumstances.

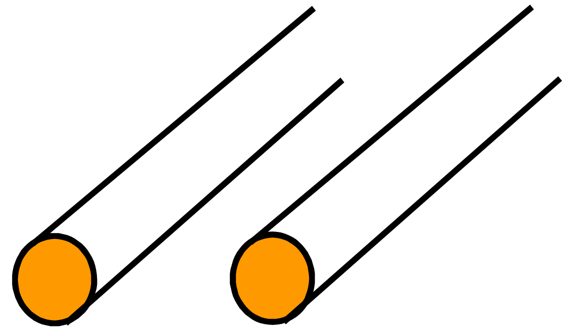


The fields are confined to the region between the two conductors.

Limitations of Transmission-Line Theory (cont.)

The twin lead is an open type of transmission line – the fields extend out to infinity.



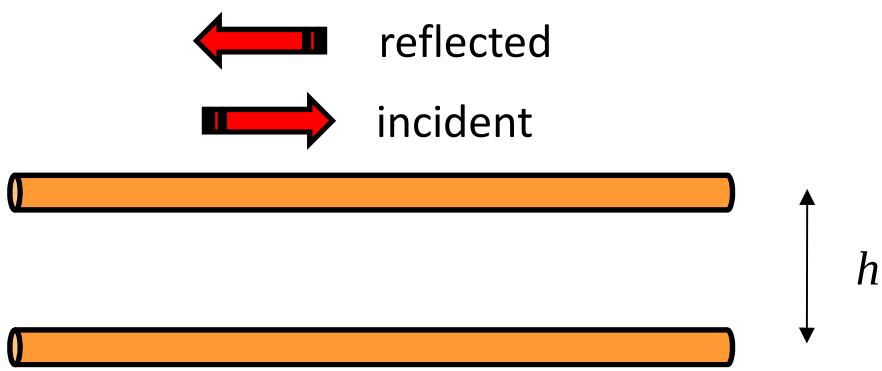


The extended fields may cause interference with nearby objects. (This may be improved by using "twisted pair.")

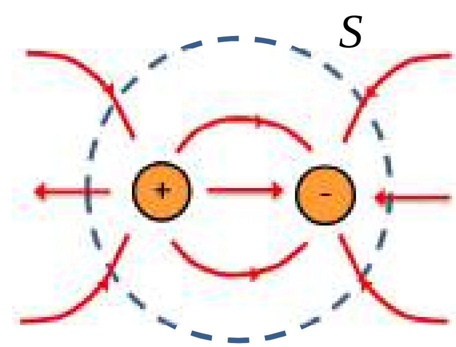
Having fields that extend to infinity is not the same thing as having radiation, however.

Limitations of Transmission-Line Theory (cont.)

The infinite twin lead will not radiate by itself, regardless of how far apart the lines are.



$$P_t = \int_S \mathrm{Re}\left(\frac{1}{2}\left(\underline{E} \times \underline{H}^*\right)\right) \cdot \underline{\hat{n}}\, dS = 0$$

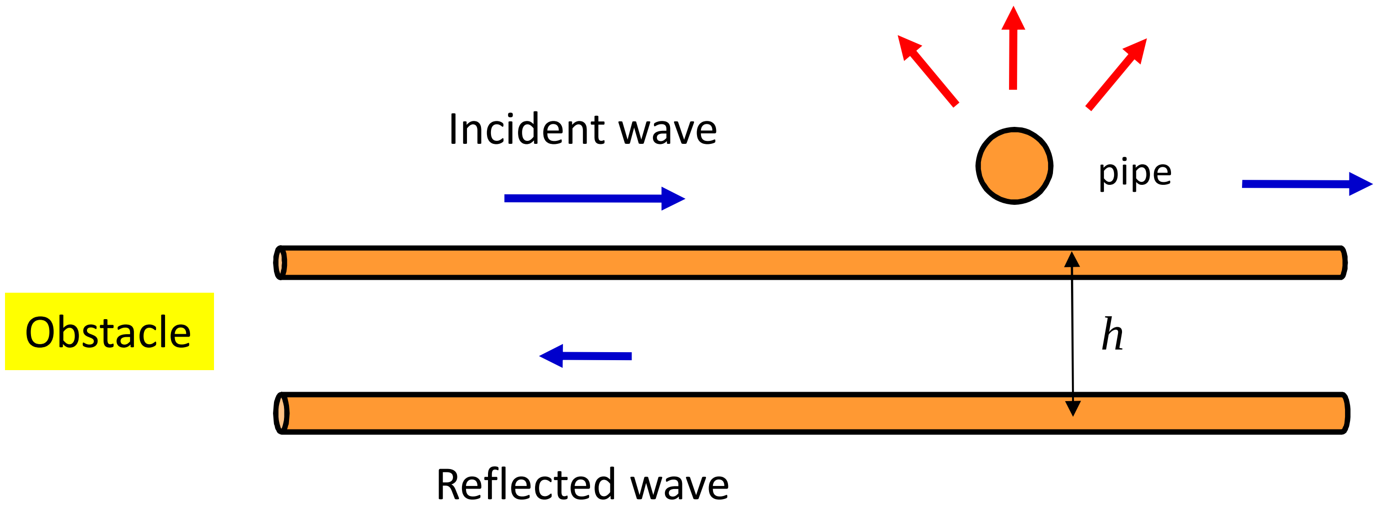


No attenuation on an infinite lossless line

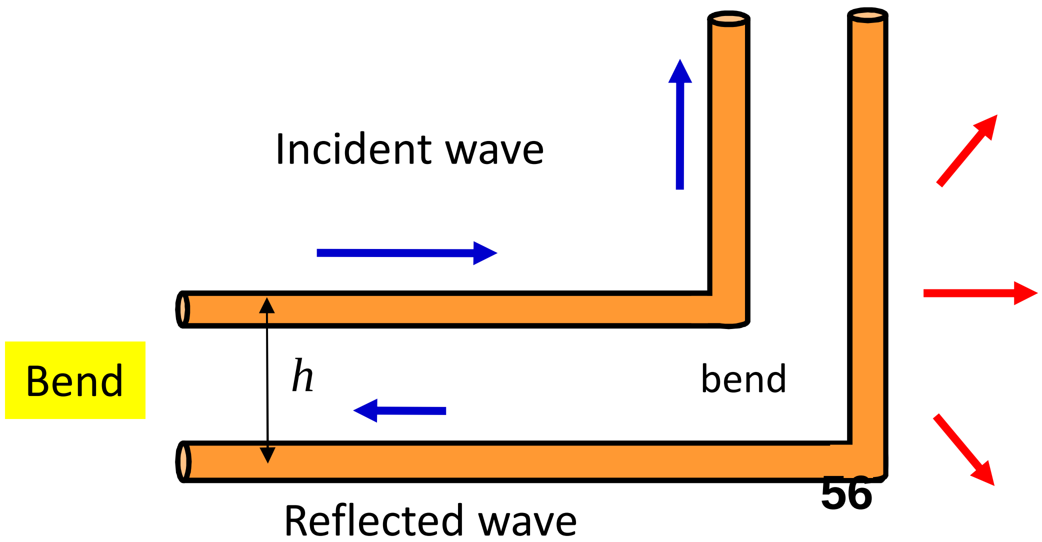
The incident and reflected waves represent an exact solution to Maxwell's equations on the infinite line, at any frequency.

Limitations of Transmission-Line Theory (cont.)

A discontinuity on the twin lead will cause radiation to occur.



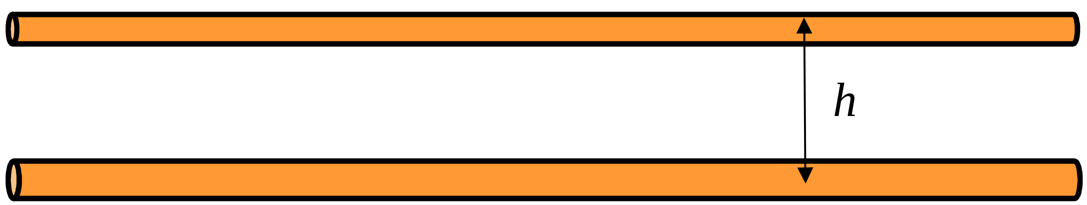
Note: Radiation effects increase as the frequency increases.



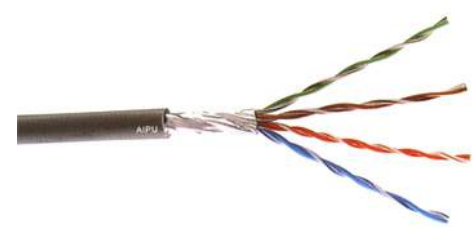
Limitations of Transmission-Line Theory (cont.)

To reduce radiation effects of the twin lead at discontinuities:

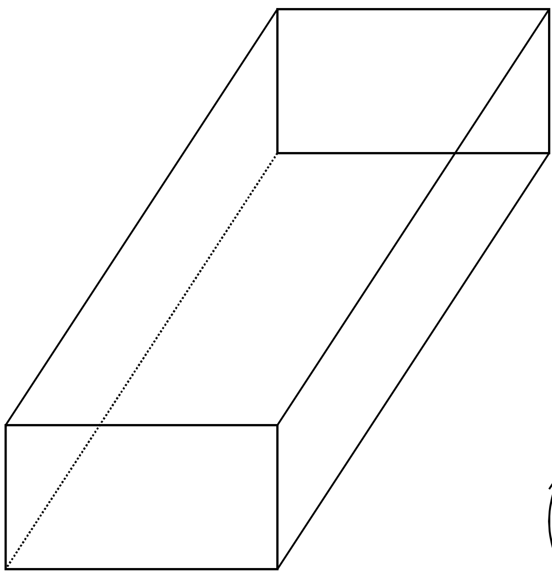
1) Reduce the separation distance h (keep $h <<$).
2) Twist the lines (twisted pair).



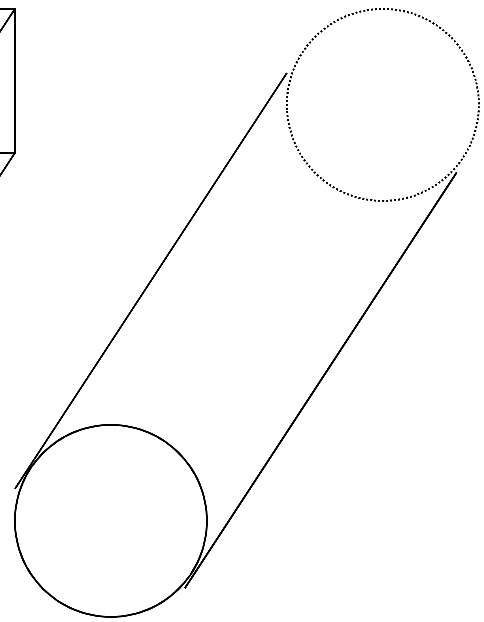
CAT 5 cable
(twisted pair)



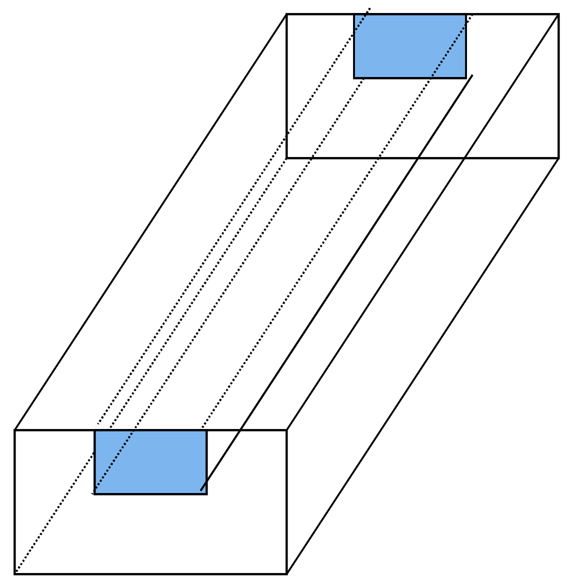
Common Hollow-pipe waveguides



Rectangular guide



Circular guide



Ridge guide

TRANSMISSION MEDIA

- **TRANSVERSE ELECTROMAGNETIC (TEM):**
  - **COAXIAL LINES**
  - **MICROSTRIP LINES (Quasi TEM)**
  - **STRIP LINES AND SUSPENDED SUBSTRATE**
- **METALLIC WAVEGUIDES:**
  - **RECTANGULAR WAVEGUIDES**
  - **CIRCULAR WAVEGUIDES**
- **DIELECTRIC LOADED WAVEGUIDES**

ANALYSIS OF WAVE PROPAGATION ON THESE TRANSMISSION MEDIA THROUGH MAXWELL'S EQUATIONS

Auxiliary Relations:

1. $\overline{F} = q\left[\overline{E} + \overline{v} \times \overline{B}\right]$ Newton

   $q \equiv$ Charge ; $\overline{v} \equiv$ Velocity

2. $\overline{J} = \text{-}\overline{E}$ (Ohm's Law)

   $\pm \equiv$ Conductivity ; $\overline{J} =$ Conduction Current

3. $\overline{J} = \pm\overline{v}$; $\overline{J} =$ Convection Current

4. $\overline{D} = £\overline{E} = £_r £_o \overline{E}$; $£_o = 8.854 \times 10^{-12}$ F/m

   $£_r \equiv$ Relative Dielectric Constant

5. $\overline{B} = «\overline{H} = «_r «_o \overline{H}$; $«_o = 4® \times 10^{-12}$ H/m

   $«_r \equiv$ Relative Permeability

Maxwell’s Equations in Large Scale Form

$$\oint_S \overline{D} \bullet \overline{d}\overline{S} = \int_V \circ \, dv$$

$$\oint_S \overline{B} \bullet \overline{d}\overline{S} = 0$$

$$\oint_l \overline{E} \bullet \overline{d}\overline{l} = -\frac{\partial}{\partial t}\int_S \overline{B} \bullet \overline{d}\overline{S} - \int_S \overline{M} \bullet d\overline{S}$$

$$\oint_l \overline{H} \bullet \overline{d}\overline{l} = \int_S \overline{J} \bullet \overline{d}\overline{S} + \frac{\partial}{\partial t}\int_S \overline{D} \bullet \overline{d}\overline{S}$$

•Maxwell’s Equations for the Time - Harmonic Case

$$\text{Assume } \mathrm{e}^{j\mu t} \text{variations, then:}$$

$$\overline{E}(x,y,z,t) = \mathrm{Re}[\overline{E}(x,y,z)e^{j\mu t}]$$

$$\overline{E}(x,y,z) = \overline{a}_x(E_{xr} + jE_{xi}) + \overline{a}_y(E_{yr} + jE_{yi})$$

$$+\overline{a}_z(E_{zr} + jE_{zi})$$

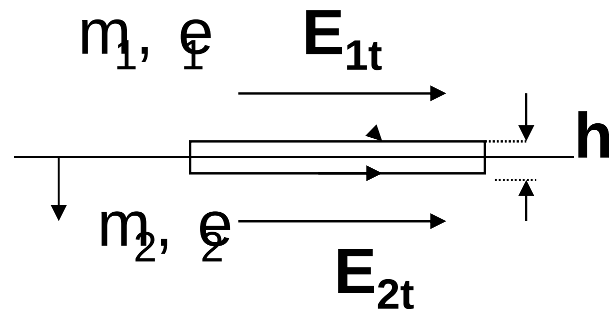
$$E_x = \mathrm{Re}[(E_{xr} + jE_{xi})e^{j\mu t}] = \mathrm{Re}[\sqrt{E_{xr}^2 + E_{xi}^2}\, e^{j\mu t + j¤}]$$

$$= \sqrt{E^2{}_{xr} + E^2{}_{xi}}\cos(\mu t + ¤) \quad , ¤ = \tan^{-1}(E_{xi}/E_{xr})$$

$$\nabla \bullet \overline{D} = ° \qquad , \nabla \bullet \overline{B} = 0$$

$$\nabla \times \overline{E} = -j\mu\overline{B} - \overline{M} \qquad , \nabla \times \overline{H} = \overline{J} + j\mu\overline{D}$$

Boundary Conditions at a General Material Interface



$$E_{t_1} - E_{t_2} = -M_s$$

$$D_{n_1} - D_{n_2} = \circ_s$$

$$\circ_s \equiv \text{Surface Charge Density}$$

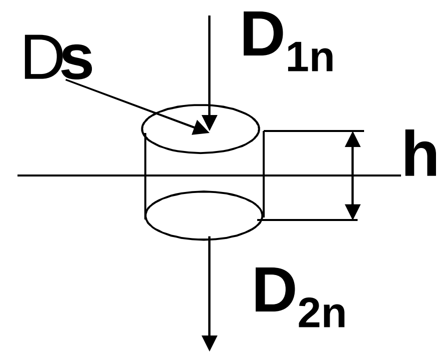
$$B_{n_1} = B_{n_2} \quad ; \quad H_{t_1} - H_{t_2} = J_s$$

$$\hat{n} \bullet (\overline{D}_1 - \overline{D}_2) = \circ_s$$

$$\hat{n} \bullet (\overline{B}_1 - \overline{B}_2) = 0$$

$$\hat{n} \times (\overline{E}_1 - \overline{E}_2) = -\overline{M}_s$$

$$\hat{n} \times (\overline{H}_1 - \overline{H}_2) = \overline{J}_s$$



Fields at a Dielectric Interface

$$E_{t_1} - E_{t_2} = 0$$

$$D_{n_1} - D_{n_2} = 0$$

$$B_{n_1} = B_{n_2} \qquad ; \qquad H_{t_1} - H_{t_2} = 0$$

$$\hat{n} \bullet (\overline{D}_1) = \hat{n} \bullet (\overline{D}_2)$$

$$\hat{n} \bullet (\overline{B}_1) = \hat{n} \bullet (\overline{B}_2)$$

$$\hat{n} \times (\overline{E}_1) = \hat{n} \times (\overline{E}_2)$$

$$\hat{n} \times (\overline{H}_1) = \hat{n} \times (\overline{H}_2)$$

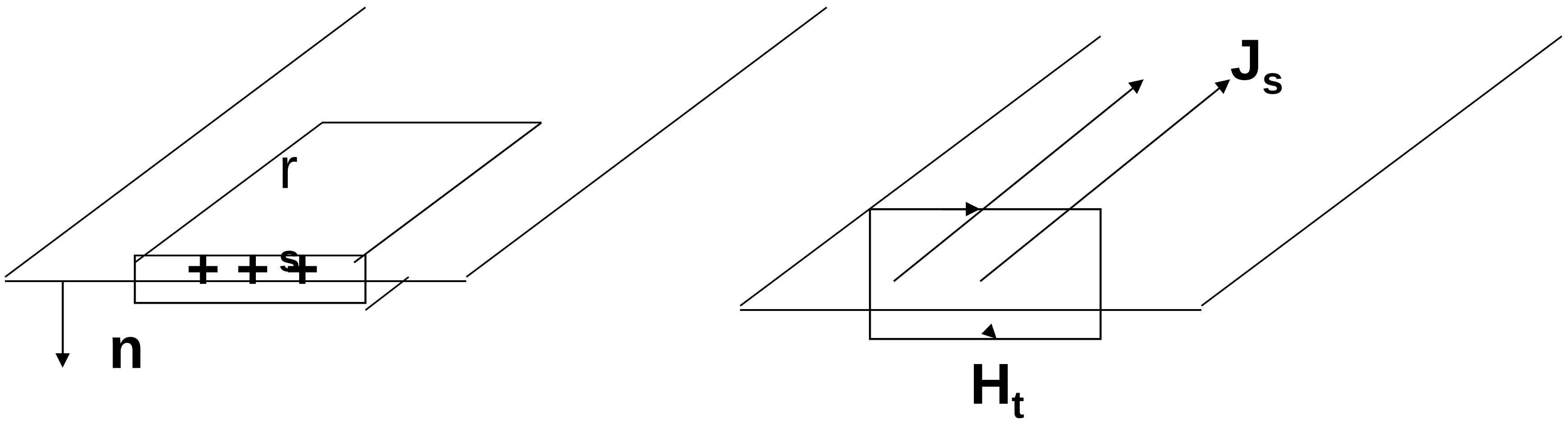
Boundary Conditions at a Perfect Conductor:

$$E_t = o \qquad \hat{n} \times \overline{E} = 0$$

$$D_n = \rho_s = \hat{n} \bullet \overline{D}$$

$$B_n = 0 \qquad \hat{n} \bullet \overline{B} = 0$$

$$J_s = H_t = \hat{n} \times \overline{H}$$



The magnetic wall boundary condition

$$\hat{n} \bullet (\overline{D}) = 0$$

$$\hat{n} \bullet (\overline{B}) = 0$$

$$\hat{n} \times (\overline{E}) = -\overline{M}_s$$

$$\hat{n} \times (\overline{H}) = 0$$

Wave Equation

$$\nabla \times \nabla \times \overline{E} = -\nabla^2 \overline{E} + \nabla(\nabla \bullet \overline{E}) = -j\mu\text{«}\nabla \times \overline{H} =$$

$$-j\mu\text{«}(\overline{J} + j\mu\pounds\overline{E})$$

For a Source free medium :

$$\nabla^2 \overline{E} + k^2 \overline{E} = 0 \qquad ; \qquad k^2 = \mu^2\text{«}\pounds$$

$$\nabla^2 \overline{H} + k^2 \overline{H} = 0 \qquad ; \qquad k = \mu / v = \frac{2\text{®}}{\text{a}}$$

Plane Waves

$$\nabla^2 \overline{E} + k_0^2 \overline{E} = 0 = \frac{\partial^2 \overline{E}}{\partial x^2} + \frac{\partial^2 \overline{E}}{\partial y^2} + \frac{\partial^2 \overline{E}}{\partial z^2} + k_0^2 \overline{E}$$

$$\frac{\partial^2 E_i}{\partial x^2} + \frac{\partial^2 E_i}{\partial y^2} + \frac{\partial^2 E_i}{\partial z^2} + k_0^2 E_i = 0 \qquad , i = x, y, z$$

Solve for $E_x(x, y, z)$, Using separation of variables $\Rightarrow$

$$\mathrm{k}_{\mathrm{x}}^2 + k_y^2 + k_z^2 = k_0^2$$

$$E_x = Ae^{-jk_s x - jk_y y - jk_z z} \qquad , \text{Let } \vec{k} = \widehat{a}_x k_x + \widehat{a}_y k_y + \widehat{a}_z k_z$$

$$\vec{r} = \widehat{a}_x x + \widehat{a}_y y + \widehat{a}_z z$$

$$E_x = Ae^{-j\bar{k}\bullet\bar{r}}, \qquad \text{Similarly } E_y = Be^{-j\bar{k}\bullet\bar{r}}, \qquad E_z = Ce^{-j\bar{k}\bullet\bar{r}}$$

$$\bar{E} = \bar{E}_0 e^{-j\bar{k}\bullet\bar{r}} \qquad \text{Since } \nabla \bullet \bar{E} = 0 \Rightarrow \bar{k} \bullet \bar{E}_0 = 0$$

The vector $\bar{E}_0$ is perpendicular to the direction of propagation $\bar{\mathrm{k}}$.

The solution is called plane wave

$$\nabla \times \bar{E} = -j\omega\mu_0 \bar{H}$$

$$\bar{H} = -\frac{1}{j\omega\mu_0}\nabla \times \bar{E}_0 e^{-j\bar{k}\bullet\bar{r}} = \frac{1}{j\omega\mu_0}\bar{E}_0 \times \nabla e^{-j\bar{k}\bullet\bar{r}} = \frac{1}{\omega\mu_0}\bar{k} \times \bar{E}_0 e^{-j\bar{k}\bullet\bar{r}}$$

$$= \frac{k_0}{\omega\mu_0}\bar{n} \times \bar{E} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\bar{n} \times \bar{E} = Y_0 \bar{n} \times \bar{E} = \frac{1}{\eta_0}\bar{n} \times \bar{E}$$

η_0 is the interensic impedance of free space $= 377\Omega$

Y_0 is the intrinsic admittance of free space.

Plane Wave in a Good Conductor

$$\pm >> \mu \text{£}$$

$$\text{¥} = \sim + j \cong j\mu\sqrt{\text{«£}}\sqrt{\frac{\pm}{j\mu\text{£}}} = (1+j)\sqrt{\frac{\mu\text{«}\pm}{2}}$$

$$\text{¢}_s = \frac{1}{\sim} = \sqrt{\frac{2}{\mu\text{«}\pm}}$$

$$\text{¦} = \frac{j\mu\text{«}}{\text{¥}} \cong (1+j)\sqrt{\frac{\mu\text{«}}{2\pm}} = (1+j)\frac{1}{\pm\text{¢}_s}$$

Boundary Conditions at the Surface of a Good Conductor

The field amplitude decays exponentially from its surfac
According to e^{-u/d_s} where u is the normal distance into th
Conductor, d_s is the skin depth

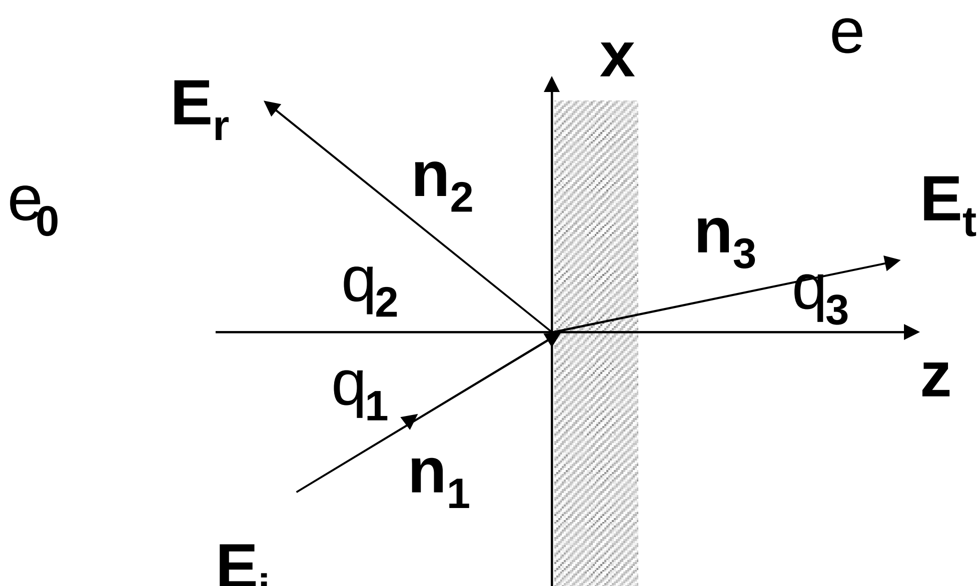
$$¢_s = \sqrt{\frac{2}{\mu«±}} \quad , \qquad \bar{J} = ±\bar{E}$$

The surface Impedance:

$$Z_m = \frac{1+j}{±¢_s}, \qquad E_t = Z_m J_s = Z_m \; \hat{n} \times \bar{H}$$

Reflection From A Dielectric Interface

Parallel Polarization



$$\overline{E}_i = \overline{E}_1 e^{-jk_0\overline{n}_1 \bullet \overline{r}_1} \quad , \quad \overline{H}_i = Y_0 \overline{n}_1 \times \overline{E}_i$$

$$\overline{E}_r = \overline{E}_2 e^{-jk_0\overline{n}_2 \bullet \overline{r}_1} \quad , \quad \overline{H}_r = Y_0 \overline{n}_2 \times \overline{E}_r$$

$$k_0 = \mu\sqrt{«_0 £} \quad , \quad Y_0 = \sqrt{\frac{£_0}{«_0}}$$

$$\overline{E}_t = \overline{E}_3 e^{-jk\ \overline{n}_3 \bullet \overline{r}_1} \quad , \quad \overline{H}_t = Y\ \overline{n}_3 \times \overline{E}_t$$

$$Y = nY_0 \quad , \quad k = \text{¦}\, k_0 \quad , n = \sqrt{\text{£}}$$

$$k_0 n_{1x} = k_0 n_{2x} = k n_{3x} = n k_0 n_{3x}$$

$$\overline{n}_1 = \hat{a}_x \sin\theta_1 + \hat{a}_z \cos\theta_1$$

$$\overline{n}_2 = \hat{a}_x \sin\theta_2 + \hat{a}_z \cos\theta_2$$

$$\overline{n}_3 = \hat{a}_x \sin\theta_3 + \hat{a}_z \cos\theta_3$$

$$\theta_1 = \theta_2 \qquad \sin\theta_1 = n \sin\theta_3$$

$$E_{1x} = E_1 \cos\theta_1 \quad , E_{1z} = -E_1 \sin\theta_1$$

$$E_{2x} = E_2 \cos\theta_2 \quad , E_{1z} = E_2 \sin\theta_2$$

$$E_{3x} = E_3 \cos\theta_3 \quad , E_{1z} = -E_3 \sin\theta_3$$

Energy and Power

Under steady-state sinusoidal time-varying Conditions, the time-average energy stored in th Electric field is

$$W_e = \mathrm{Re}\frac{1}{4}\int_V \overline{E} \bullet \overline{D}^* dV = \frac{1}{4}\int_V £'\overline{E} \bullet \overline{E}^* dV$$

If £ is constant and real, then

$$W_e = \frac{£}{4}\int_V \overline{E} \bullet \overline{E}^* dV$$

Time average energy stored in the magnetic field is :

$$W_m = \mathrm{Re}\frac{1}{4}\int_V \overline{H}^* \bullet \overline{B}dV$$

$$= \frac{\text{«}}{4}\int_V \overline{H} \bullet \overline{H}^* dV \quad \text{if « is real and constant}$$

The time average power transmitted across a closed surface S is given by :

$$P = \frac{1}{2}\mathrm{Re}\oint_S \overline{E} \times \overline{H}^* \bullet dS$$

Poynting Theorem

$$\nabla \bullet \overline{E} \times \overline{H}^* = (\nabla \times \overline{E}) \bullet \overline{H}^* - (\nabla \times \overline{H}^*) \bullet \overline{E}$$

$$= (-j\mu \overline{B} - \overline{M}) \bullet \overline{H}^* + j\mu \overline{D}^* \bullet \overline{E} - \overline{E} \bullet \overline{J}^*$$

$$\overline{J} = \overline{J}_s + \pm \overline{E}$$

$$-\frac{1}{2}\int_V \nabla \bullet \overline{E} \times \overline{H}^* dV = -\frac{1}{2}\oint_S \overline{E} \times \overline{H}^* \bullet dS$$

$$= j\frac{\mu}{2}\int_V (\vec{B} \bullet \overline{H}^* - \overline{E} \bullet \overline{D}^*) dV + \frac{1}{2}\int_V (\overline{E} \bullet \overline{J}^* + \overline{H}^* \bullet \overline{M}_s) dV$$

$$= 2j\mu \int_V \left(\frac{\overline{B} \bullet \overline{H}^*}{4} - \frac{\overline{E} \bullet \overline{D}^*}{4} \right) dV + \frac{1}{2}\int_V (\overline{E} \bullet \overline{J}^* + \overline{H}^* \bullet \overline{M}_s) dV$$

If the medium is characteri zed by :

$$£ = £' - j£'', \qquad « = «' - j«'' \quad \text{and conductivi ty } ±$$

$$- \frac{1}{2}\int_V (\overline{E} \bullet \overline{J}_{Ss} + \overline{H}^* \bullet \overline{M}_S)dV = \frac{1}{2}\oint_S \overline{E} \times \overline{H}^* \bullet dS +$$

$$\frac{1}{2}\int_V ±\overline{E} \bullet \overline{E}^* dV + \frac{μ}{2}\int_V («''\overline{H} \bullet \overline{H}^* + £''\overline{E} \bullet \overline{E}^*)dV +$$

$$j\frac{μ}{2}\int_V («'\overline{H} \bullet \overline{H}^* - £'\overline{E} \bullet \overline{E}^*)dV$$

$$P_0 = \frac{1}{2}\oint_S \overline{E} \times \overline{H}^* \bullet dS$$

$$P_\ell = \frac{\mu}{2}\int_V (\text{«}''\overline{H}\bullet\overline{H}^* + \text{£}''\overline{E}\bullet\overline{E}^*)dV + \frac{1}{2}\int_V \pm\overline{E}\bullet\overline{E}^* dV$$

$$= \text{Time average power loss}$$

$$\mathrm{P_s} = -\frac{1}{2}\int_V (\overline{E}\bullet\overline{J}_s + \overline{H}_s^*\bullet\overline{M}_s)dV$$

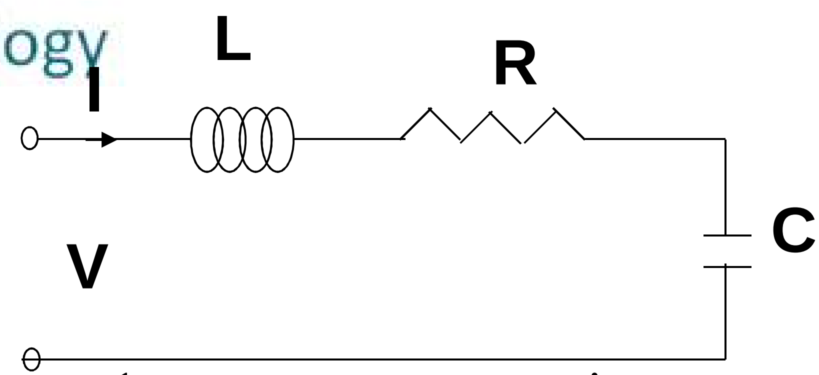
$$-\,\mathrm{Im}\frac{1}{2}\oint_S \overline{E}\times\overline{H}^*\bullet dS = 2\mu\int_V\left(\text{«}'\frac{\overline{H}\bullet\overline{H}^*}{4} - \text{£}'\frac{\overline{E}\bullet\overline{E}^*}{4}\right)dV$$

$$= 2\mu(W_m - W_e)$$

$$P_s = P_0 + P_\ell + 2j\mu(W_m - W_e)$$

The power delivered by the sources ($\mathrm{P_s}$) is equal to the sum of the power transmitted through the surface $\mathrm{P_0}$, the power lost to heat in the volume (P_ℓ) and 2μ times the reactive energy stored in the volume.

Circuit Analogy



$$\frac{1}{2}VI^* = \frac{1}{2}ZII^* = \frac{1}{2}II^*(R + j\mu L - \frac{j}{\mu C})$$

$$= \frac{1}{2}RII^* + 2j\mu(\frac{1}{4}LII^* - \frac{1}{4}\frac{II^*}{\mu^2 C})$$

$$= P_\ell + 2j\mu(W_m - W_e)$$

$$Z = \frac{P_\ell + 2j\mu(W_m - W_e)}{\frac{1}{2}II^*}$$

General Definitio n of the impedance of a network

Transmission Lines & Waveguides

Wave Propagation in the Positive z-Direction is Represented By: $e^{-j\beta z}$

$$\overline{E}(x,y,z) = \overline{E}_t(x,y,z) + \overline{E}_z(x,y,z)$$

$$= \overline{e}_t(x,y)e^{-j\beta z} + \overline{e}_z(x,y)e^{-j\beta z}$$

$$\overline{H}(x,y,z) = \overline{H}_t(x,y,z) + \overline{H}_z(x,y,z)$$

$$= \overline{h}_t(x,y)e^{-j\beta z} + \overline{h}_z(x,y)e^{-j\beta z}$$

$$\nabla \times \overline{E} = (\nabla_t - j\beta\,\overline{a}_z)\times(\overline{e}_t + \overline{e}_z)e^{-j\beta z} = -j\mu\omega(\overline{h}_t + \overline{h}_z)e^{-j\beta z}$$

$$\nabla_t \times \overline{e} - j\beta\,\overline{a}_z \times \overline{e} + \nabla_t \times \overline{e}_z - j\beta\,\overline{a}_z \times \overline{e}_z = -j\mu\omega(\overline{h}_t + \overline{h}_z)e^{-j\beta z}$$

$$\nabla_t \times \overline{e}_t = -j\mu\omega\overline{h}_z \qquad , \nabla_t \times \overline{h}_t = j\mu\varepsilon\overline{e}_z$$

$$\nabla_t \bullet \overline{h}_t = j\beta\,\overline{h}_z \qquad , \nabla_t \bullet \overline{e}_t = j\beta\,\overline{e}_z$$

Modes Classification:

1. Transverse Electromagnetic (TEM) Waves

$$E_z = H_z = 0$$

2. Transverse Electric (TE), or H Modes

$$E_z = 0 \quad , \quad \text{but } H_z \neq 0$$

3. Transverse Magnetic (TM), or E Modes

$$H_z = 0 \quad , \quad \text{But } \ E_z \neq 0$$

4. Hybrid Modes

$$H_z \neq 0 \quad , \quad E_z \neq 0$$

TEM WAVES

$$\nabla_t \bullet \bar{h}_t = 0 \qquad , \qquad \nabla_t \bullet \bar{e}_t = 0$$

$$\nabla_t \times \bar{e}_t = 0 \qquad , \qquad \hat{a}_z \times \bar{e}_t = \mu\text{«}_0 \bar{h}_t$$

$$\nabla_t \times \bar{h}_t = 0 \qquad , \qquad \hat{a}_z \times \bar{h}_t = -\mu\text{£}_0 \bar{e}_t$$

$$\bar{e}(x, y) = -\nabla_t \Phi(x, y) = 0 \qquad \Phi \equiv \text{Scalar Potential}$$

$$\therefore \nabla_t^{\,2} \Phi(x, y) = 0$$

$$\bar{E}_t = \bar{e}_t e^{\mp j\ z} = -\nabla_t \Phi(x, y) e^{\mp j\ z}$$

$$\bar{H}_t = \pm \bar{h}_t e^{\mp j\ z} = \pm Y_0 \hat{a}_z \times \bar{e} e^{\mp j\ z}$$

$$Y_0 = \sqrt{\frac{£}{«}} = \frac{1}{Z_0} \quad , \quad ¦_0 = \text{Wave Impedance}$$

$$\frac{\mathrm{E_x}}{\mathrm{H_y}} = -\frac{E_y}{H_x} = \pm ¦_0$$

$\pm$ for wave propagation in the + or - z direction

The field must satisfy Helmholtz equation :

$$\nabla^2 E_t + k_0^2 E_t = 0 \quad , \quad \text{but } \nabla = \nabla_{\mathrm{t}} - j\,¯\,a_z \qquad , \nabla^2 = \nabla_t^2 - \quad {}^2$$

$$\nabla_t^2 \overline{E}_t + (k_0^2 - \quad)\overline{E}_t = 0 \qquad , \qquad \nabla_{\mathrm{t}}\,[\nabla_t^2 ¤ + (k_0^2 - \quad)¤] = 0$$

$$= \pm k_0 \quad \text{for TEM waves}$$

TE WAVES

$$\nabla^2 \overline{H} + k^2 \overline{H} = 0$$

$$(\nabla_t^{\,2} - \quad{}^2)h_z(x, y) + k^2 h_z = 0$$

$$\nabla_t^{\,2} + (k^2 - \quad{}^2)h_z = 0, \qquad \text{let } k_c^2 = k^2 - \quad{}^2$$

$$\nabla_t^{\,2} h_z + k_c^2 h_z = 0$$

$$\nabla_t \times \overline{e}_t = -j\mu\text{«}h_z \qquad , \qquad \overline{a}_z \times \overline{e}_t = \mu\text{«}\overline{h}_t$$

$$\nabla_t \times \overline{h}_t = 0 \qquad , \qquad \overline{a}_z \times \nabla_t h_z + j \quad a_z \times \overline{h}_t = -j\mu\text{£}\overline{e}$$

$$\nabla_t \bullet \overline{h}_t = j \quad h_z \qquad , \qquad \nabla_t \bullet \overline{e}_t = 0$$